Estimates of black-hole parameters in XTE J1650-500 and GRS 1915+105 from the Extended Orbital Resonance Model for high-frequency QPOs

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Abstract: Humpy profile of the LNRF-related orbital velocity was found for Keplerian discs orbiting near-extreme Kerr black holes with dimensionless spin $a > 0.9953$ (Aschenbach 2004). The hump in orbital velocity occurs close to but above the innermost stable circular orbit (ISCO) in the disc, see Fig. 1. The maximal positive rate of change of the orbital velocity with respect to the proper radial distance is used to determine locally defined frequency characterizing any processes in the disc capable to excite hypothetical, so-called "humpy", oscillations which are proposed to be connected with local non-monotonic behaviour of the orbital velocity. Comparing the "humpy frequency" related to distant observers with the radial epicyclic frequency we found that for very rapid Kerr black holes ($a \gtrsim 0.998$) the ratio of radial epicyclic frequency $\nu_{\rm r}$ and humpy frequency $\nu_{\rm h}$ is in terms of small integers asymptotically going to the ratio ~ 3:2 for $a \to 1$. The Extended Orbital Resonance Model (ExORM) suggests an excitation of epicyclic oscillations by the velocity hump in the inner part of the disc close to ISCO, especially if the resonant phenomena are in the game. This model can be aplied only for near-extreme black-hole candidates in which high-frequency QPOs have been observed. The ratio of frequencies determines the spin (and vice versa), and the values of frequencies determine the black-hole mass.

XTE J1650-500 is a Galactic black-hole binary system for which at least one high-frequency QPO at 250 Hz has been reported. Moreover, there are indications that the system harbours a near-extreme Kerr black hole with a spin $a \simeq 0.998$ and mass $M_{\rm BH} \lesssim 7.3\,M_\odot$. For the Kerr black hole with spin $a = 0.9982$ the humpy frequency and the radial epicyclic frequency are in ratio 1:3 at orbit where positive rate of change of the LNRF-related orbital velocity with proper radial distance is maximal. Identifying the radial epicyclic frequency with the observed 250 Hz QPO, we obtain for the mass of the black hole value $5.1\,M_\odot$ (Slaný & Stuchlík 2008).

GRS 1915+105 is a Galactic black-hole binary system for which at least five high-frequency QPOs at frequencies 27 Hz, 41 Hz, 67 Hz, 113 Hz, and 167 Hz have been reported. There are indications that the system harbours a near-extreme Kerr black hole with a spin $a > 0.98$ and mass $M_{\rm BH} = (14.0 \pm 4.4)\,M_\odot$. Identifying the radial epicyclic frequency and the humpy frequency with the observed 67 Hz and 41 Hz QPOs, respectively, we arrive at the black-hole parameters $M_{\rm BH} = 14.8\,M_\odot$ and $a = 0.9998$ (Stuchlík et al. 2007b).

Orbital velocity

In the Kerr spacetime, which gives the general relativistic description of a gravitational field around rotating black hole, we can introduce a natural set of local observers – so called ZAMO (Zero Angular Momentum Observers) who, in some sense, rotate with the geometry. The related frames are called LNRF (Locally Non-Rotating Frames). Each observer carries an orthonormal tetrad of 1-forms

$$\mathbf{e}^{(t)} = \left(\frac{\Sigma\Delta}{A}\right)^{1/2}\mathbf{d}t, \quad \mathbf{e}^{(\varphi)} = \left(\frac{A}{\Sigma}\right)^{1/2}\sin\theta\,(\mathbf{d}\varphi - \omega\,\mathbf{d}t), \tag{1}$$

$$\mathbf{e}^{(r)} = \left(\frac{\Sigma}{\Delta}\right)^{1/2}\mathbf{d}r, \qquad \mathbf{e}^{(\theta)} = \Sigma^{1/2}\mathbf{d}\theta, \tag{2}$$

where

$$\Delta = r^2 - 2Mr + a^2, \qquad \Sigma = r^2 + a^2\cos^2\theta, \tag{3}$$

$$A = (r^2 + a^2)^2 - a^2\Delta\sin^2\theta, \qquad \omega = 2ar/A, \tag{4}$$

and, rather than describe physical quantities by their coordinate components at each point, one gives their projections onto the tetrad, i.e., their physically measured components. The local frame of any physical observer differs from the LNRF at observer's location only by a Lorentz transformation. In our description we use geometrized units ($c = G = 1$) and Boyer-Lindquist coordinates (t, r, θ, φ). Further we put $M = 1$ to obtain completely dimensionless formulae hereafter.

For matter with a 4-velocity $U^\mu = (U^t, 0, 0, U^\varphi)$ and angular velocity $\Omega = U^\varphi/U^t$ orbiting the Kerr black hole, the orbital velocity is given by the azimuthal component of its 3-velocity in the LNRF

$$\mathcal{V}^{(\varphi)} = \frac{U^\mu \mathrm{e}^{(\varphi)}_\mu}{U^\nu \mathrm{e}^{(t)}_\nu} = \frac{A\sin\theta}{\Sigma\sqrt{\Delta}}(\Omega - \omega); \tag{5}$$

ω, given by (3), is the angular velocity of the LNRF relative to distant observers.

In thin (Keplerian, geodesical) discs the matter follows nearly circular equatorial geodesics characterized by Keplerian distribution of angular velocity

$$\Omega = \Omega_{\rm K}(r;\, a) \equiv \frac{1}{(r^{3/2} + a)}. \tag{6}$$

The orbital velocity of thin discs is given by the relation

$$\mathcal{V}^{(\varphi)}(r;\, a) = \frac{(r^2 + a^2)^2 - a^2\Delta - 2ar(r^{3/2} + a)}{r^2(r^{3/2} + a)\sqrt{\Delta}} \tag{7}$$

and, as found by Aschenbach (2004), it has a "humpy profile" in the field of rapidly rotating Kerr black holes with $a > 0.9953$ (see Fig. 1).

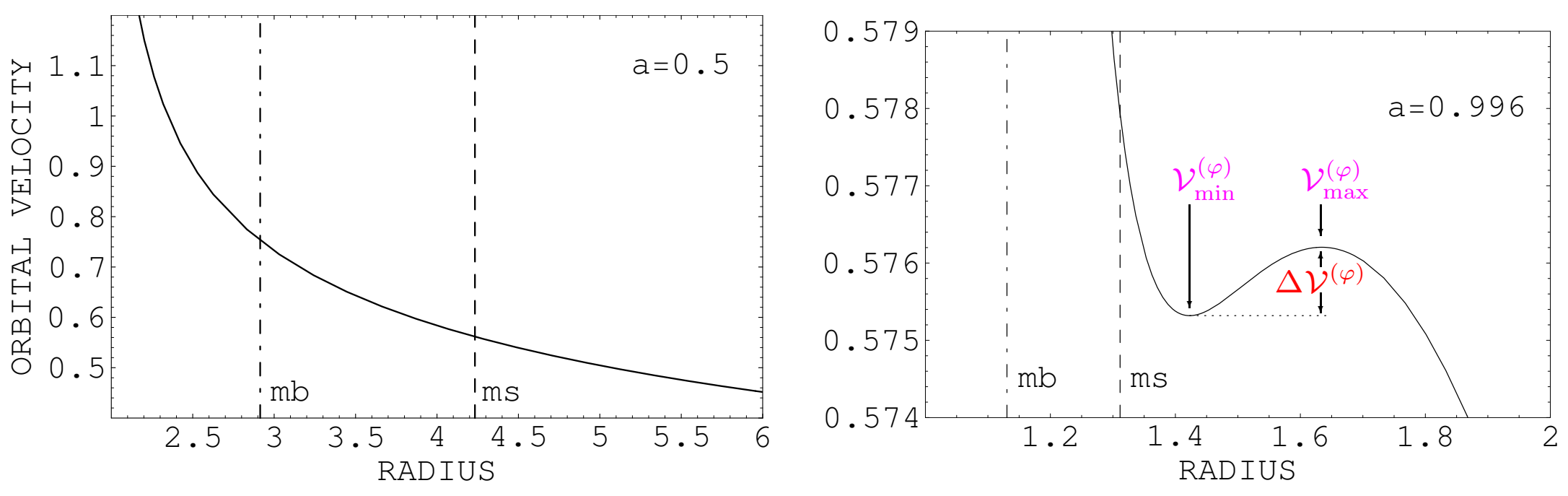


Fig. 1: Orbital velocity profiles for non-extreme and near-extreme Kerr black holes. Dashed (dashed-dotted) line determines location of the marginally stable (marginally bound) orbit.

Humpy oscillations and ExORM

Aschenbach (2004) proposed an excitation of oscillations in inner part of accretion discs around very rapidly rotating black holes due to non-monotonic behaviour of the orbital velocity in the disc. Since the hump is located close to the ISCO, it could be related to high-frequency QPOs observed in some near-extreme Kerr black hole candidates.

Critical frequency of hump-induced oscillations is defined as the maximal positive rate of change of the orbital velocity $\mathcal{V}^{(\varphi)}$ in terms of the proper radial distance $\tilde{r}$ in order to give coordinate-indepentent characteristic frequency quantifying Aschenbach's effect (Stuchlík et al. 2007a)

$$\nu_{\rm crit} = \left.\frac{\partial\mathcal{V}^{(\varphi)}}{\partial\tilde{r}}\right|_{\rm max}, \quad \mathbf{d}\tilde{r} = \sqrt{g_{rr}}\mathbf{d}r = \sqrt{\frac{\Sigma}{\Delta}}\mathbf{d}r. \tag{8}$$

Relating this locally defined characteristic frequency to distant observers, we obtain so-called "humpy frequency"

$$\nu_{\rm h} = \sqrt{-(g_{tt} + 2\omega g_{t\varphi} + \omega^2 g_{\varphi\varphi})}\,\nu_{\rm crit}; \tag{9}$$

$g_{\mu\nu}$ are metric coefficients of the Kerr geometry. Humpy frequency $\nu_{\rm h}$ represents an upper limit on characteristic frequencies of proposed oscillations. "Realistic" humpy frequency, as observed by distant observer, can be expected close to but smaller than that given by relation (9). Due to possible changes of the humpy frequency, Aschenbach's effect in the disc should not be characterized by a narrow frequency peak but rather by one relatively wide and not very strong QPO with a centroid frequency of several tens Hz for stellar mass black holes.

In the Orbital Resonance Model (ORM) of Abramowicz & Kluźniak the attention is payed to basic orbital frequencies connected with perturbed geodesical motion, i.e. to the radial and vertical epicyclic frequencies

$$\nu_{\rm r}^2 = \nu_{\rm K}^2(1 - 6r^{-1} + 8ar^{-3/2} - 3a^2r^{-2}), \tag{10}$$

$$\nu_{\rm v}^2 \equiv \nu_\theta^2 = \nu_{\rm K}^2(1 - 4ar^{-3/2} + 3a^2r^{-2}), \tag{11}$$

where $\nu_{\rm K} = \Omega_{\rm K}/2\pi$. The Extended ORM suggests a resonant excitation of epicyclic motion in the inner part of the disc by the orbital velocity hump. Due to existence of strong gravitational field near the ISCO, non-linear resonant phenomena are expected. In this case, oscillations with combinational frequencies $\nu_{\rm r} \pm \nu_{\rm h}$ are also possible.

$1 - a$	$\nu_{\rm crit}$	$\nu_{\rm h}$
4.5×10^{-3}	121	29
4×10^{-3}	432	102
3×10^{-3}	1130	248
1×10^{-3}	3061	536
5×10^{-4}	3789	592
2×10^{-4}	4352	**607**
1×10^{-4}	4579	603
1×10^{-5}	4816	590
1×10^{-6}	4841	588
1×10^{-9}	4844	588

ν are in units of $(M/M_\odot)^{-1}$ Hz.

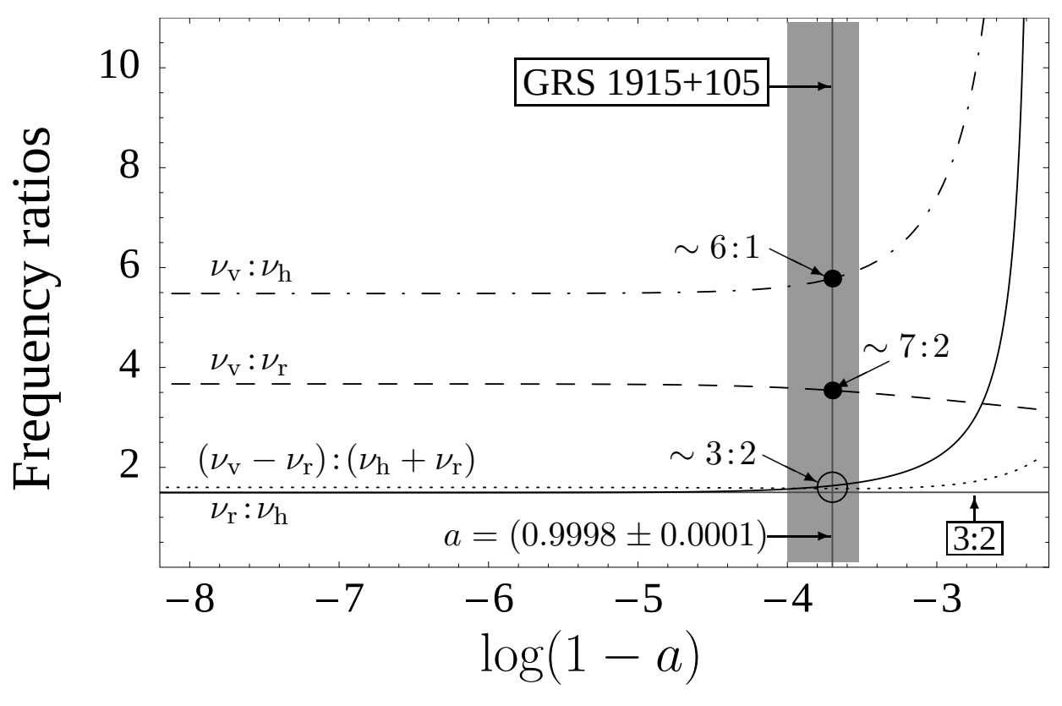


Fig. 2: Spin dependence of frequency ratios. Range of the spin relevant for GRS 1915+105 is shaded.

XTE J1650-500

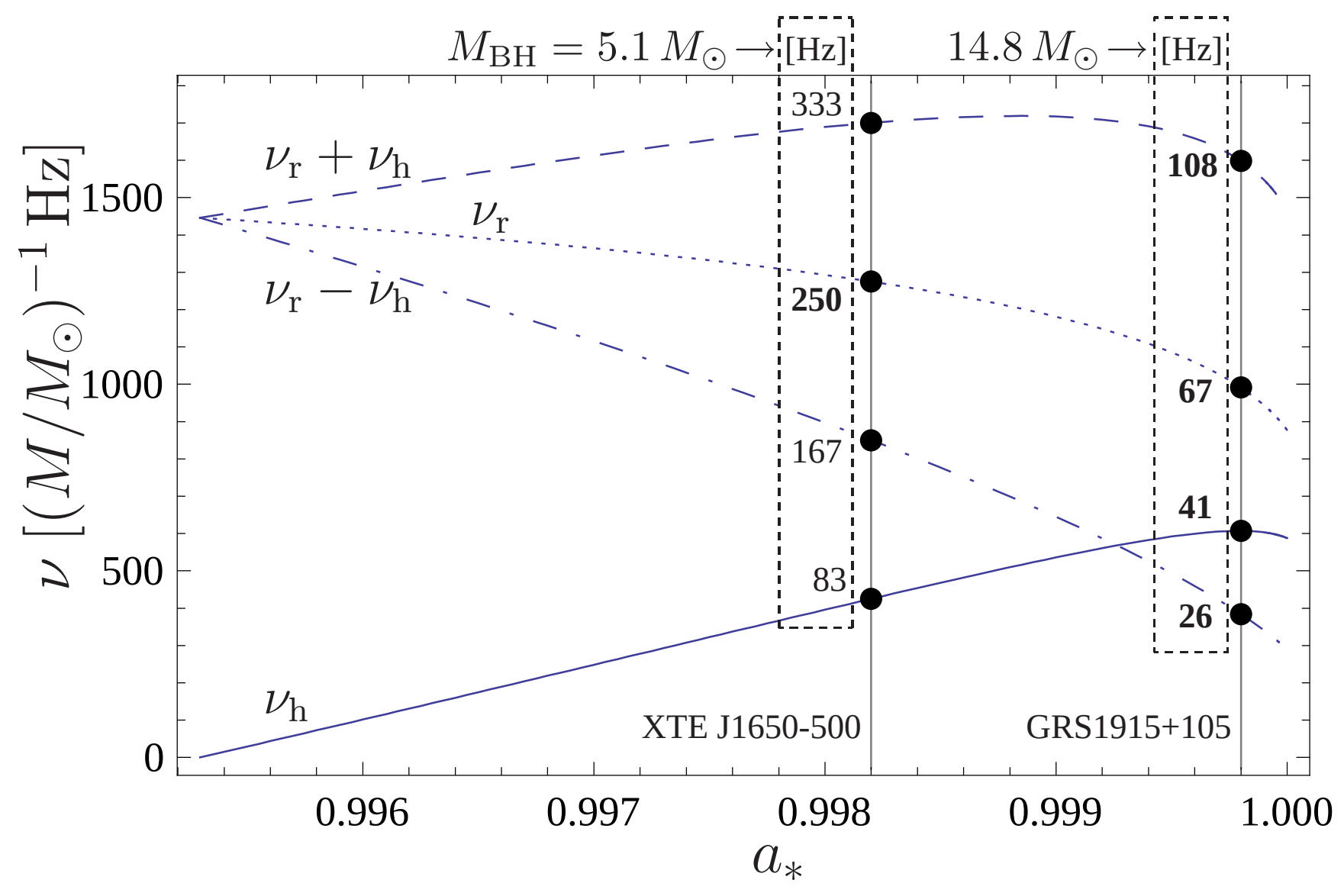
XTE J1650-500 is a near-extreme Kerr black hole candidate with spin $a \simeq 0.998$ inferred from the broad, skewed Fe Kα line first reported by Miller et al. (2002). Homan et al. (2003) report detection of HFQPO around 250 Hz and other broad high-frequency features around 50 Hz, 109 Hz, and 168 Hz. Kalemci et al. (2003) report another broad features around 80 Hz and 25 Hz but no QPO.

For spin $a = 0.9982$ the ratio $\nu_{\rm r}$:$\nu_{\rm h}$, evaluated at the radius determining $\nu_{\rm crit}$, is very close to 3:1. Identifying the observed 250 Hz QPO with the radial epicyclic frequency, which for such spin and resonant radius (given by the model) is $1275\,(M/M_\odot)^{-1}$ Hz, we obtain an estimate of the black-hole mass following from the ExORM as $5.1\,M_\odot$, which is well below the upper limit of $7.3\,M_\odot$ given by the optical mass-function (Orosz et al. 2004).

GRS 1915+105

GRS 1915+105 is another near-extreme Kerr black hole candidate with spin $a > 0.98$ following from analysis of its spectral continuum performed by McClintock et al. (2006). Moreover, at least five HFQPOs with frequencies 27 Hz (Belloni et al. 2001), 41 Hz, 67 Hz, 113 Hz, and 168 Hz (Remillard & McClintock 2006) were observed in this source.

Suggesting the 41 Hz QPO to be the humpy frequency $\nu_{\rm h}$ and 67 Hz QPO the radial epicyclic frequency $\nu_{\rm r}$ at the same orbit, we obtain the spin $a = 0.9998$. Note that for this spin the humpy frequency in units of black-hole mass reaches its maximal value $607\,(M/M_\odot)^{-1}$ Hz. Mass estimate following from the ExORM is therefore $14.8\,M_\odot$. Moreover, the 113 Hz and 27 Hz QPOs can correspond to combinational frequencies $\nu_{\rm r} \pm \nu_{\rm h}$. The 168 Hz QPO is very close to the difference between the vertical and radial epicyclic frequencies $\nu_{\rm v} - \nu_{\rm r}$, again both computed at the same radius as the humpy frequency, or to $4\nu_{\rm h}$.



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