kHz QPOs in discs orbiting neutron stars testing braneworld models

Andrea Kotrlová, Zdeněk Stuchlík and Gabriel Török

Institute of Physics, Faculty of Philosophy and Science, Silesian University in Opava, Bezručovo nám. 13, CZ-746 01 Opava
e-mail: *Andrea.Kotrlova@fpf.slu.cz, Zdenek.Stuchlik@fpf.slu.cz, terek@volny.cz*

http://www.physics.cz

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Introduction

One of the most promising approaches to the higher-dimensional gravity theories seems to be the string theory and M-theory describing gravity as higher-dimensional interaction becoming effectively 4D at low enough energies and these theories inspired braneworld models, where the observable universe is a 3-brane (domain wall) to which the standard non-gravitational matter fields are confined, while gravity field enters the extra spatial dimensions, the size of which may be much larger than the Planck length scale $l_{\mathrm{P}} \sim 10^{-33}\,\mathrm{cm}$.
The strong gravitational field of neutron stars in the braneworld universe of the Randall–Sundrum type II [1] with infinite additional dimension could be described by spherically symmetric solutions with metric in the exterior to the braneworld stars being of the Reissner–Nordström (RN) type containing a braneworld tidal charge representing the tidal effect of the bulk spacetime onto the star structure. We investigate the role of the tidal charge in orbital models of high-frequency quasiperiodic oscillations (QPOs) observed in neutron star binary systems. We show how the standard relativistic precession (RP) model modified by the tidal charge fits the observational data, giving estimates of the allowed values of the tidal charge and the brane tension based on the processes going in the vicinity of neutron stars. We compare our strong field regime restrictions with those given in the weak field limit of solar system experiments.

Braneworld neutron stars

There are two known solutions of the effective 4D Einstein equations corresponding to the braneworld uniform density configurations that could roughly represent the spacetime exterior to neutron stars [2]. We focus our attention on the solution of the RN type that is formally identical to the spherically symmetric black hole solution, however, its parameters are related to the brane tension λ and energy density ρ of the internal configuration due to the matching conditions of the internal and external spacetimes. The external spacetime is given in the standard Schwarzschild coordinates by the line element

$$\mathrm{d}s^2 = -A(r)\mathrm{c}^2\mathrm{d}t^2 + A^{-1}(r)\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\phi^2, \qquad A(r) = 1 - \frac{2r_{\mathrm{G}}}{r} + \frac{B}{r^2}, \qquad r_{\mathrm{G}} = \frac{\mathrm{G}M}{\mathrm{c}^2}, \tag{1}$$

where M is the gravitational mass and B is the braneworld tidal charge representing the non-local bulk effects on the 4D spacetime structure. The tidal charge B stands instead of the charge parameter Q^2 of the standard RN spacetimes; B can be both positive and negative, but there are indications that $B < 0$ should properly represent the effects of the bulk space Weyl tensor [3]. The matching conditions imply [2]

$$B = -\frac{3\mathrm{G}M}{\mathrm{c}^2}R\left(\frac{\rho}{\lambda}\right), \qquad M = \widetilde{M}\left(1 - \frac{\rho}{\lambda}\right), \tag{2}$$

where R is the internal configuration radius. Clearly, negative (positive) values of B correspond to positive (negative) values of the brane tension λ.
The effective mass and the effective density are given by

$$\widetilde{M} = 4\pi\int_0^R \rho^{\mathrm{eff}}(r)r^2\mathrm{d}r, \qquad \rho^{\mathrm{eff}} = \rho\left(1 + \frac{\rho}{2\lambda}\right) + \frac{6}{(8\pi\mathrm{G})^2\lambda}\mathcal{U}. \tag{3}$$

The non-local bulk gravitational effects arising from the bulk Weyl tensor are represented by the "dark energy" density $\mathcal{U}$. The standard general relativistic equations are regained in the limit $\lambda^{-1} \to 0$. The internal solution puts two important restrictions on the brane tension λ [2]:

$$\lambda \geq \left(\frac{\mathrm{G}M/\mathrm{c}^2}{R - 2\mathrm{G}M/\mathrm{c}^2}\right)\rho, \qquad \frac{\mathrm{G}M/\mathrm{c}^2}{R} \leq \frac{4}{9}\left[\frac{1 + 5\rho/4\lambda}{(1 + \rho/\lambda)^2}\right]. \tag{4}$$

The first one is general for all uniform stars and implies $R > 2\mathrm{G}M/\mathrm{c}^2$, i.e., the Schwarzschild radius is still the relevant limit as in general relativity. The second one represents an upper limit on compactness of the star following from the requirement that pressure must be finite inside the star. It implies the general relativistic limit $\left(\mathrm{G}M/\mathrm{c}^2\right)/R \leq 4/9$ for $\lambda^{-1} \to 0$. We can see that the braneworld high-energy corrections reduce the compactness limit of the star.

Circular geodesics of the external spacetime

Considering vacuum spacetimes, the event horizons of the braneworld RN metric are determined by the condition $A(r) = 0$. Using geometric units $(\mathrm{c} = \mathrm{G} = 1)$, the radius of the outer event horizon is given by the relation

$$r_+ = M + \sqrt{M^2 - B}\,. \tag{5}$$

The horizon structure depends on the sign of the tidal charge B. For $B < 0$ it follows from equation (5) that the horizon radius $r_+ > 2M$; such a situation is not allowed in the framework of general relativity [3]. Introducing the dimensionless radial coordinate x and the dimensionless braneworld parameter b

$$x = \frac{r}{r_{\mathrm{G}}}, \qquad b = \frac{B}{r_{\mathrm{G}}^2}, \tag{6}$$

the radii of the outer event horizon $x_{\mathrm{h}}(b)$, the circular photon orbit $x_{\mathrm{ph}}(b)$, the marginally bound orbit $x_{\mathrm{mb}}(b)$ and the marginally stable orbit $x_{\mathrm{ms}}(b)$ are implicitly determined by the relations

$$b = b_{\mathrm{h}}(x) \equiv x\,(2 - x)\,, \qquad b = b_{\mathrm{ph}}(x) \equiv \frac{x}{2}\,(3 - x)\,, \qquad b = b_{\mathrm{mb}}(x) \equiv x\left(2 \mp \sqrt{x}\right), \qquad b = b_{\mathrm{ms}}(x) \equiv \frac{x}{8}\left(9 \mp \sqrt{16x - 15}\right). \tag{7}$$

We have to put the limits on the applicability of the vacuum spacetime using the limit condition on the compactness of the uniform internal configuration (4) that can be expressed in the form

$$4 - \left(9 + \frac{5}{3}b\right)\left(\frac{r_{\mathrm{G}}}{R}\right) + 6b\left(\frac{r_{\mathrm{G}}}{R}\right)^2 - b^2\left(\frac{r_{\mathrm{G}}}{R}\right)^3 \geq 0\,. \tag{8}$$

The equality in the relation (8) determines the critical radius R_{C} representing limit on the radius of neutron stars. We have to restrict our studies to the region $r > R_{\mathrm{C}}$. The radii $X_{\mathrm{C}} = R_{\mathrm{C}}/r_{\mathrm{G}}$ are given (in an implicit form) by the relation

$$b = b_{\mathrm{C}}(x) \equiv \frac{1}{6}\left(18x - 5x^2 \mp x^{3/2}\sqrt{25x - 36}\right). \tag{9}$$

The functions $b_{\mathrm{h}}(x)$, $b_{\mathrm{ph}}(x)$, $b_{\mathrm{mb}}(x)$, $b_{\mathrm{ms}}(x)$ and $b_{\mathrm{C}}(x)$ are illustrated in Fig. 1. It is evident that the positive tidal charge will play the same role in its effect on the circular orbits as the electric charge in the RN spacetime – the radius of the circular photon orbit, as well as the radii of the marginally bound and the marginally stable circular orbits shift inwards as the positive tidal charge increases. For the negative tidal charge the limiting photon orbit, the marginally bound and the marginally stable circular orbits shift outwards as the absolute value of b increases [3].

Epicyclic oscillations of Keplerian motion

It is well known that for oscillations of both thin Keplerian and toroidal discs around neutron stars (black holes) the orbital Keplerian frequency ν_{K} and the related radial and vertical epicyclic frequencies ν_{r} and ν_θ of geodetical quasicircular motion are relevant and observable directly or through some combinational frequencies. In the case of the external braneworld neutron star spacetime of the RN type with the braneworld tidal charge b, the formulae of the test particle geodetical circular motion and its epicyclic oscillations, obtained in [4], could be directly applied (putting $a = 0$ and $Q^2 = b$):

$$\nu_{\mathrm{K}} = \frac{1}{2\pi}\left(\frac{\mathrm{G}M}{r_{\mathrm{G}}^3}\right)^{1/2}\frac{1}{x^{3/2}}\left(1 - \frac{b}{x}\right)^{1/2} = \frac{1}{2\pi}\frac{\mathrm{c}^3}{\mathrm{G}M}\frac{1}{x^{3/2}}\left(1 - \frac{b}{x}\right)^{1/2}, \tag{10}$$

$$\nu_{\mathrm{r}}^2 = \alpha_{\mathrm{r}}\nu_{\mathrm{K}}^2, \quad \text{where} \quad \alpha_{\mathrm{r}} = \left(1 - \frac{b}{x}\right)^{-1}\left(1 - \frac{6}{x} + \frac{9b}{x^2} - \frac{4b^2}{x^3}\right), \tag{11}$$

$$\nu_\theta^2 = \alpha_\theta\nu_{\mathrm{K}}^2, \quad \text{where} \quad \alpha_\theta = 1, \tag{12}$$

so that $\nu_{\mathrm{K}}(x, b) = \nu_\theta(x, b)$ due to the spherical symmetry of the spacetime.
The Keplerian and radial epicyclic frequency profiles are illustrated in Fig. 1. For the RN type spacetimes with $b \leq 1$, the radial epicyclic frequency ν_{r} has a local maximum located at $x = \mathcal{X}_{\mathrm{r}} > x_{\mathrm{ms}}$, and vanishes at the marginally stable circular geodesic. The Keplerian frequency has a local maximum located at $x = \mathcal{X}_{\mathrm{K}} = 4b/3$. Generally, the maximum of the Keplerian frequency is physically irrelevant for all the braneworld RN neutron stars with $b \leq 1$, since it could never be located above the circular photon orbit x_{ph} (and the marginally stable orbit x_{ms}). A detailed discussion of the properties of the Keplerian and radial epicyclic frequency considering also the RN naked singularity type spacetimes is given in [5].
The role of the tidal charge in the orbital resonance model of QPOs in discs around braneworld Kerr black holes was recently studied in [6].

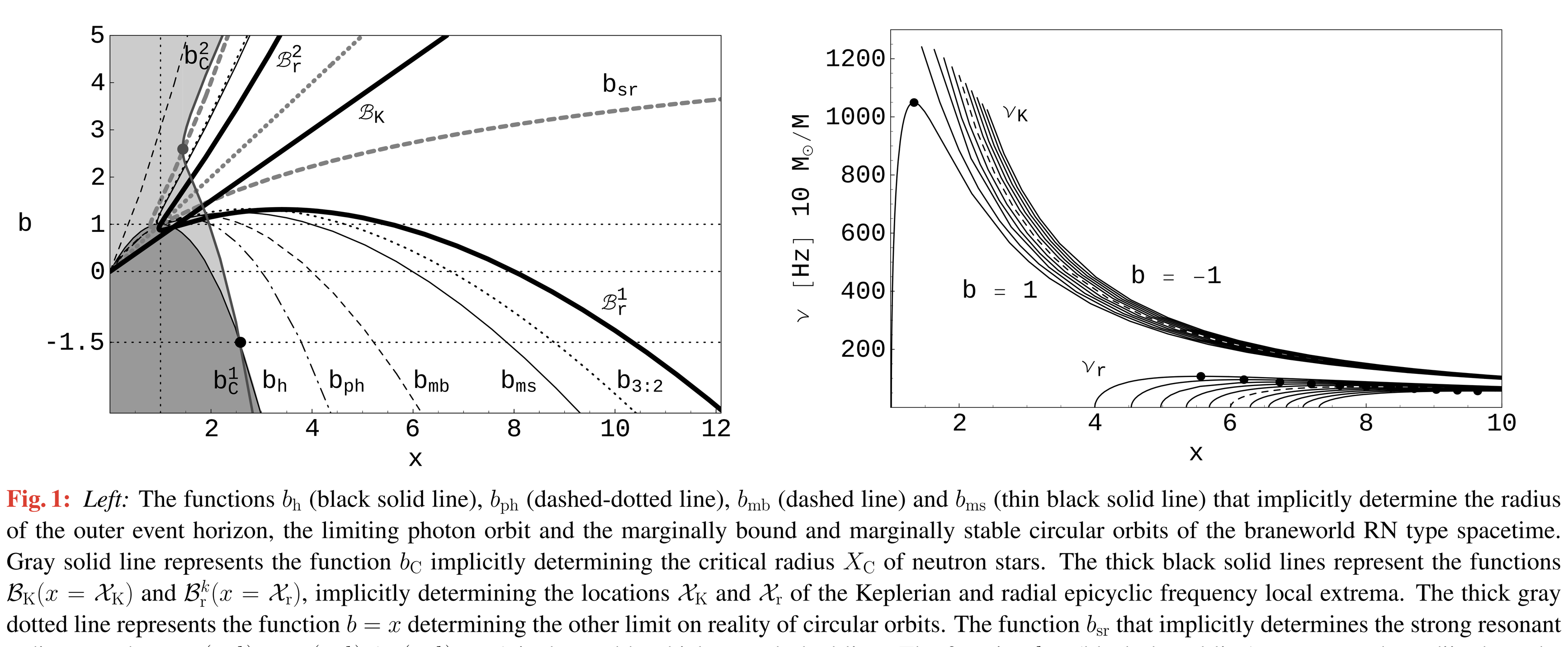

Fig. 1: *Left:* The functions b_{h} (black solid line), b_{ph} (dashed-dotted line), b_{mb} (dashed line) and b_{ms} (thin black solid line) that implicitly determine the radius of the outer event horizon, the limiting photon orbit and the marginally bound and marginally stable circular orbits of the braneworld RN type spacetime. Gray solid line represents the function b_{C} implicitly determining the critical radius X_{C} of neutron stars. The thick black solid lines represent the functions $\mathcal{B}_{\mathrm{K}}(x = \mathcal{X}_{\mathrm{K}})$ and $\mathcal{B}_{\mathrm{r}}^k(x = \mathcal{X}_{\mathrm{r}})$, implicitly determining the locations $\mathcal{X}_{\mathrm{K}}$ and $\mathcal{X}_{\mathrm{r}}$ of the Keplerian and radial epicyclic frequency local extrema. The thick gray dotted line represents the function $b = x$ determining the other limit on reality of circular orbits. The function b_{sr} that implicitly determines the strong resonant radius x_{sr} where $\nu_{\mathrm{K}}(x, b) = \nu_{\mathrm{r}}(x, b)$ $(\alpha_{\mathrm{r}}(x, b) = 1)$ is denoted by thick gray dashed line. The function $b_{3:2}$ (black dotted line) represents the radii where the relativistic precession resonance $\nu_{\mathrm{K}}\!:\!(\nu_{\mathrm{K}} - \nu_{\mathrm{r}}) = 3\!:\!2$ occurs. *Right:* The behaviour of the Keplerian frequency ν_{K} and radial epicyclic frequency ν_{r} in the field of braneworld RN neutron star spacetimes with various values of the tidal charge parameter b. The curves are spaced by 0.2 in b and they are plotted from the outer event horizon x_{h}. The dashed lines represent Schwarzschild spacetime with zero tidal charge.

Orbital models of QPOs

A number of low mass X-ray binaries containing a neutron star show QPOs in their X-ray flux, i.e., peaks in the Fourier variability power density spectra (PDS). Frequencies of some QPOs are in the kHz range corresponding to frequencies of the orbital motion close to the central neutron star. Such a so called high-frequency (kHz) QPOs span large frequency range of $200 - 1400$ Hz. The two distinct modes following their own correlation between frequency and properties (amplitude and quality factor) of the peak are observed in this range (see, e.g., [7, 8]). They are called lower and upper QPO because the frequencies of upper QPO, ν_{U}, is higher than the frequency of the lower QPO, ν_{L}, when both modes are detected simultaneously; we call them twin peak QPOs in such situations. The twin peak QPOs are clustered around frequencies corresponding to the frequency ratio of small integers, indicating thus relevance of some resonant phenomena [9, 10, 11, 12].
Several models have been outlined to explain observational data of the neutron star kHz QPOs assuming that their origin is related to the orbital motion near the inner edge of accretion discs around the neutron stars. The RP model of Stella and Vietri [13] introduces the QPOs representing a direct manifestation of modes of a relativistic epicyclic motion of radiating blobs in the marginally parts of the accretion disc. In this model, the upper and lower frequencies are identified in the following way:

$$\nu_{\mathrm{U}} = \nu_{\mathrm{K}}, \qquad \nu_{\mathrm{L}} = \nu_{\mathrm{K}} - \nu_{\mathrm{r}}. \tag{13}$$

The resonance model introduced by Abramowicz and Kluźniak [14] (and earlier in other connections by Aliev and Galtsov [4]) assumes a nonlinear resonance (forced or parametric) of accretion disc oscillations with vertical and radial epicyclic frequencies ν_θ and ν_{r}. In spherically symmetric spacetimes assumed here $\nu_\theta = \nu_{\mathrm{K}}$, and we identify

$$\nu_{\mathrm{U}} = \nu_{\mathrm{K}}, \qquad \nu_{\mathrm{L}} = \nu_{\mathrm{r}}. \tag{14}$$

These models are valid at $x > x_{\mathrm{ms}}$, but their behaviour is inverse as $x \to x_{\mathrm{ms}}$, since $\nu_{\mathrm{K}}\!:\!(\nu_{\mathrm{K}} - \nu_{\mathrm{r}}) \to 1$ there, while $\nu_{\mathrm{K}}\!:\!\nu_{\mathrm{r}}$ diverges there since $\nu_{\mathrm{r}}(x = x_{\mathrm{ms}}) = 0$. The observed frequency ratio $\nu_{\mathrm{U}}/\nu_{\mathrm{L}}$ decreases with increasing frequencies as $x \to x_{\mathrm{ms}}$ but $\nu_{\mathrm{K}}/\nu_{\mathrm{r}}$ increases as $x \to x_{\mathrm{ms}}$. Frequency relations implied by the RP model yield a trend which is in good accord with observation. To model the data of twin peak QPOs in neutron star systems, we can use the RP model, but the direct epicyclic frequency model gives trends just opposite to what is observed.

QPOs and RP model testing the braneworld models

We have summarized the twin peak QPOs data for the atoll sources and Z-sources Sco X-1 and Cir X-1 in Fig. 2 using the data accumulated in [8].

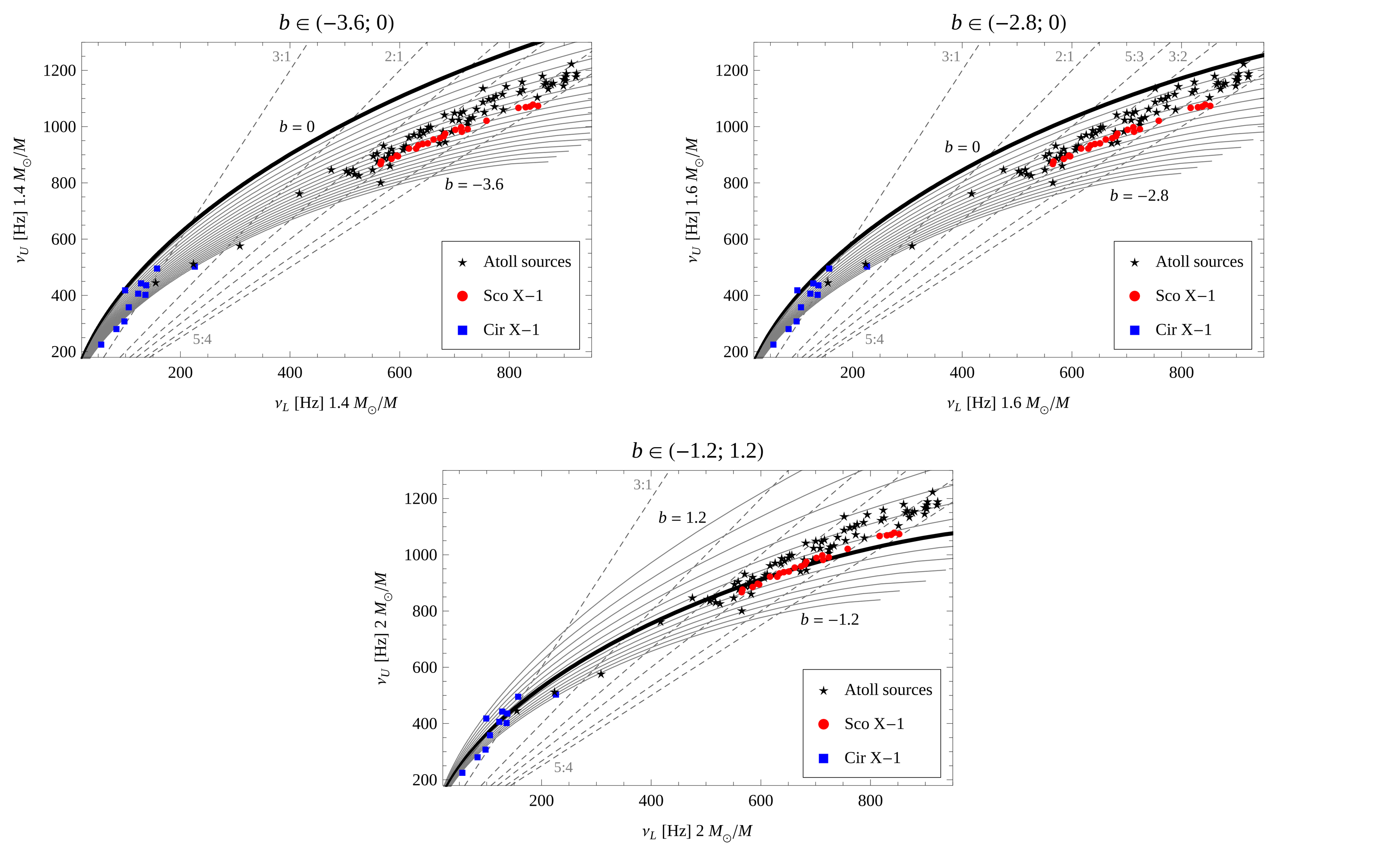

Fig. 2: The RP model fits for atoll sources, Sco X-1 and Cir X-1 for various values of braneworld parameter b; the curves are spaced by 0.2 in b. The gray dashed lines represent the frequency ratios $\nu_{\mathrm{U}} : \nu_{\mathrm{L}} = 3\!:\!1$, $2\!:\!1$, $5\!:\!3$, $3\!:\!2$, $4\!:\!3$ and $5\!:\!4$ subsequently left to right. Note that the $M = 2\,\mathrm{M}_\odot$, $b = 0$ RP frequency relation is commonly used for the fitting of observed QPO data (see, e.g., [8]).

The restrictions of the tidal charge B and the brane tension λ can be obtained by using the relations

$$|B| < |B_{(i)}| = |b_{(i)}|\left(\frac{\mathrm{G}M_{(i)}}{\mathrm{c}^2}\right)^2, \qquad |\lambda| > |\lambda_{(i)}| = \frac{3\mathrm{G}M_{(i)}}{\mathrm{c}^2}\frac{R_{(i)}\rho_{(i)}}{B_{(i)}} = \frac{3\rho_{(i)}}{|b_{(i)}|}\frac{R_{(i)}}{r_{\mathrm{G}}} = \frac{3\rho_{(i)}}{|b_{(i)}|}X_{(i)}. \tag{15}$$

In order to make a more precise restriction of λ, we have to estimate the dimensionless surface radius X of the configuration for given $b_{(i)}$ and $M_{(i)}$. We find

$$|\lambda| > |\lambda_{(i)}| = \frac{3}{|b_{(i)}|}\left(\frac{\mathrm{M}_\odot}{M_{(i)}}\right)^2\left(\frac{1}{X_{(i)}}\right)^2 1.477 \times 10^{17}\,\mathrm{g/cm}^3. \tag{16}$$

Results

$\lvert b\rvert$	$M\ [\mathrm{M}_\odot]$	$\lvert B\rvert\ [\mathrm{cm}^2]$	$\lvert\lambda\rvert\ [\mathrm{g \cdot cm^{-3}}]$
< 3.6	1.4	$< 1.537 \times 10^{11}$	$> 2.7 \times 10^{15}$ (a)
< 2.8	1.6	$< 1.562 \times 10^{11}$	$> 3.9 \times 10^{15}$ (b)
< 1.2	2.0	$< 1.046 \times 10^{11}$	$> 1.0 \times 10^{16}$ (c)

Tab. 1: Estimates on restrictions of the tidal charge B and the brane tension λ calculated (a) for the canonical values of the neutron stars with $R = 10\,\mathrm{km}$, $\rho \sim 6.6 \times 10^{14}\,\mathrm{g \cdot cm^{-3}}$ using the relations (15), and obtained by choosing (b) $X = 4$, (c) $X = 3$ in relation (16).

Conclusions

Using the RP model of kHz QPOs observed in neutron star X-ray binaries modified by the hypothetical tidal charge of the neutron stars as implied by the braneworld uniform star model, we are able to put rough restriction on the tidal charge magnitude and brane tension magnitude related to the considered group of atoll sources and some Z-sources. This is the first attempt at an estimate coming from the consideration of effects in the extremely strong gravitational fields. These values should be confronted with estimates given by Böhmer et al. [15], concerning the effects of the tidal charge in the limit of weak gravitational field related to the solar system. They considered the perihelion precession (Mercury), deflection of light near the edge of Sun and the radar echo delay experiment. Clearly, the most convenient for comparison seems to be the first phenomenon, being of the same nature as the epicyclic motion of blobs in the RP model. The weak field restrictions are [15]

$$|B| < 5 \times 10^8\,\mathrm{cm}^2, \qquad \lambda > 7 \times 10^{13}\,\mathrm{g \cdot cm^{-3}}. \tag{17}$$

Note that the rough strong field regime model gives estimates of the tidal charge higher than the weak field regime in more than two orders of magnitude. However, as stressed in [15], a substantial part of the perihelion shift could correspond to the effect of solar oblateness, modifying thus, in principle, very strongly the estimates of B and λ. On the other hand, the restrictions implied in [15] by the light deflection and radar echo delay are $|B| < 10^{19}\,\mathrm{cm}^2$ and $|\lambda| > 10^{17}\,\mathrm{g \cdot cm^{-3}}$, which are substantially, by orders, higher than those given by rough estimates of QPOs observational data in X-ray binaries. We can thus conclude that the restrictions on $|B|$ and $|\lambda|$ implied by rough fitting of the neutron star X-ray binaries kHz QPOs data, i.e., in the strong gravitational field regime, give better fits as compared to those implied by the weak field limit, if these results are not masked by the effects of the solar oblateness. Clearly, further and more detailed studies of all these phenomena are necessary. Moreover, there is a variety of other vacuum solutions of spherically symmetric static field on the brane that deserve attention and have to be tested separately since the Weyl tensor in the complete 5D theory is unknown yet.
We plan to make further, more sophisticated comparisons of the plausible kHz QPOs models for neutron star X-ray binaries, including the RP model and the resonance disc oscillation ones, and compare these models with data obtained for individual sources. Generally, we can conclude that the presence of a negative tidal charge enables lowering of the neutron star mass as can be seen from our rough estimates.

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