

## Observational Tests of Neutron Star Relativistic Mean Field Equations of State

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### ABSTRACT

Set of neutron star observational results is used to test some selected equations of state of dense nuclear matter. The first observational result comes from the mass–baryon number relation for pulsar B of the double pulsar system J 0737–3039. The second one is based on the mass–radius relation coming from observation of the thermal radiation of the neutron star RX J 1856.35–3754. The third one follows the population analysis of isolated neutron star thermal radiation sources. The last one is the test of maximum mass. The equation of state of asymmetric nuclear matter is given by the parameterized form of the relativistic Brueckner-Hartree-Fock mean field, and we test selected parameterizations that represent fits of full relativistic mean field calculation. We show that only one of them is capable to pass the observational tests. This equation of state represents the first equation of state that is able to explain all the mentioned observational tests, especially the very accurate test given by the double pulsar even if no mass loss is assumed.

**Key words:** Stars: neutron, Equations of state, Dense matter

### 1. Introduction

Neutron stars are compact objects that play important role in different areas of modern physics. Here we concentrate our attention on the possibility that phenomena related to neutron stars can be used as tests of equation of state (EoS) of asymmetric nuclear matter. The tests used in this paper represent a subset of tests used previously by Klähn *et al.* (2006). We focus our attention on tests that come from astronomical observations. However we have not applied the very promising test coming from the observations of quasiperiodic oscillations (QPOs), since the theory and data interpretation is still in progress (see e.g. Török *et al.* (2008a,b,c, 2010) van der Klis (2004)). The QPO test applied on the 4U 1636–536 object in Klähn *et al.* (2006) represents the maximum mass test in the present paper, and another test following the QPO phenomena observed in the 4U 0614+09 object do not provide a strong test and all the EoS tested in this paper pass it.

A wide spectrum of different equations of state of nuclear matter and their applications to astrophysical problems has been reported in literature (see, e.g.,

Haensel, Zdunik and Douchin 2002, Rikovska Stone *et al.* 2003, Weber, Negreiros and Rosenfeld 2007, Lattimer and Prakash 2007, Burgio 2008). Some of the EoS collections (even though not all of them are up-to-date already) give an amazingly rich general overview of the state-of-the-art, whereas the others emphasize some specific aims. All these EoS yield (nearly) the same properties close to the standard nuclear density ( $\rho_N \approx 0.16 \text{ nucleon/fm}^3 \approx 2.7 \times 10^{14} \text{ g/cm}^3$ ), but when one is far off this value, s/he has to rely more on underlying principles than on possible experimental verification of predicted physical observables.

Here we concentrate our attention on relativistic asymmetric nuclear matter where the EoS stem from an assumed form of the interaction Lagrangian. The calculations use the relativistic mean-field theory with allowance for an isospin degree of freedom (Kubis and Kutchera 1997, Müller and Serot 1996). We employed the Dirac-Brueckner-Hartree-Fock mean-field approach in its parameterized form suggested in Gmuca (1991) which reproduces the nuclear matter results of Huber, Weber and Weigel (1995). That has been used to calculate high-density behavior of asymmetric nuclear matter with varying neutron-to-proton ratio (Gmuca 1992). The proton fraction has been determined from the condition of  $\beta$ -equilibrium and charge neutrality, and it is density-dependent. We have extended our calculations for densities up to  $4 \times \rho_N$  and if there was an astrophysical motivation even higher.

The EoS is used to model the static, spherically symmetric neutron star in the framework of general relativity. The equation of hydrostatic equilibrium is solved for different central parameters (pressure, energy density, baryon number density). The radius of the neutron star model is then given by the condition of vanishing pressure. The resulting properties of the neutron star model are then compared with observational data. From the test ensemble presented by Klähn *et al.* (2006) we choose four astrophysical observations to test our selected parameterizations that have been found to be a good description of nuclear matter at subnuclear densities for pure neutron matter and up to  $2 \times \rho_N$  for symmetric nuclear matter (Kotulić Bunta and Gmuca 2003).

The maximum mass test is the standard way to test the EoS of asymmetric nuclear matter (see e.g. Haensel, Potekhin and Yakovlev 2007, Lattimer and Prakash 2007, Klähn *et al.* 2006). The usual value to constrain the maximal mass of neutron star comes from observations of double pulsar PSR 0751+1807 giving  $M = (2.1 \pm 0.2) M_\odot$ , with  $M_\odot$  being the solar mass. This value was, however, lowered to  $M = (1.26 \pm 0.14) M_\odot$  (Nice, Stairs and Kasian 2008) and could not be used as maximum mass test anymore. Another value that could serve as maximum mass test comes from the observation of QPOs. The mass is constrained on the basis that the observed frequency corresponds to the frequency at innermost stable circular orbit (Barret, Olive and Miller 2005, Belloni, Mendez and Homman 2007, van der Klis 2004).

Popov *et al.* (2006) used the population synthesis of the isolated neutron star sources of thermal radiation and concluded that the neutron stars with mass  $M <$

$1.5 M_{\odot}$  could not cool via direct URCA reactions. This conclusion follows from the fact that all observed sources of thermal radiation have masses below the quoted value. This could be explained by the fact that more massive objects cool via the direct URCA reactions which represents the fast cooling scenario and thus the thermal radiation could not be detected. These arguments were used by Klähn *et al.* (2006) to build a strong and a weak test on EoS.

A very accurate test of EoS was developed by Podsiadlowski *et al.* (2005). They based it on the model of double pulsar system J 0737–3039 formation. The model predicts the pulsar B of this system to be born via the electron capture supernova which suggests extremely low mass loss and thus the number of particles conserved during the progenitor collapse to neutron star. This puts limits on the mass–baryon number relation. Instead of the baryon number that represents the total number of baryons contained in the neutron star, the baryon mass could be used equally.

The thermal radiation coming from the neutron star source RX J1856.35–3754 could be used to put limits on the mass–radius relation of the neutron star model. Trümper *et al.* (2004) used two different models to explain the spectral feature for this specific source and found its apparent radius that represents the radius of the neutron star as seen by a distant observer. The analyses of data to obtain the isolated neutron star radius strongly depend on the radiation spectrum emitted by the object and the estimated radius is proportional to the distance from Earth to the source. The distances obtained for RX J1856.5–3754 range from  $D = 61_{-8}^{+9}$  pc (Walter and Matthews 1997) to  $D = 161_{-14}^{+18}$  pc (van Kerkwijk and Kaplan 2007). The derived apparent radius  $R_{\infty}$  is given by the model of the atmosphere. The original model by Pons *et al.* (2002) resulted in  $R_{\infty}/D = 0.13 \text{ km.pc}^{-1}$ . Trümper *et al.* (2004) presented new models of atmosphere leading to the estimates of  $R_{\infty} = 16.5 \text{ km}$  for the two component model of spectra and  $R_{\infty} = 16.8 \text{ km}$  assuming continuous temperature distribution model. If the distance derived by van Kerkwijk and Kaplan (2007) and the original model of Pons *et al.* (2002) are used together, they lead to an unexpectedly high estimate  $R_{\infty} = 20.9 \text{ km}$ . Recently Steiner, Lattimer and Brown (2010) presented results based on new analysis of data giving the distance  $119 \pm 5 \text{ pc}$  and the original model for atmosphere (Pons *et al.* 2002) then implies  $R_{\infty} = 15.47 \text{ km}$ . We decided to use the three values  $R_{\infty} = 15.5, 16.8, 20.9 \text{ km}$  to put limits on neutron star equation of state.

Another promising way to constrain the equation of state are the moment of inertia measurements (see e.g. Lattimer and Prakash 2007 and references therein). Two ways have been proposed quite recently. One for the Crab pulsar (Bejger and Haensel 2002,2003) following observations of the pulsar–nebula system, and the other for the pulsar A of the double pulsar system J0737–3039 (Bejger, Bulik and Haensel 2005) based on the measurements of the second order post Newtonian parameters of the binary system. Even though both ways could provide strong limits on the equations of state in principle, they need more accurate observational inputs. We need better estimates of the mass of Crab nebula in the first case and

very accurate measurements of orbital parameters are necessary to calculate the moment of inertia in the second case. For these reasons we do not include these tests to our calculations. The measurements of moment of inertia of the neutron star together with its mass put limits on the radius of the neutron star that is crucial for the cooling scenarios (see e.g. Lattimer and Prakash 2007, Stuchlík *et al.* 2009).

The paper is organized as follows. In section 2 we present our EoS and details of the neutron star matter description. Section 3 briefly summarizes the model of static spherically symmetric neutron star. We present our results and compare them to observations in section 4. The paper is closed by conclusions in section 5.

## 2. Equation of state of neutron star matter

### 2.1. Asymmetric nuclear matter in relativistic mean-field approach

We follow the Dirac-Brueckner-Hartree-Fock (DBHF) mean field (see Weber 1999, Walecka 2004, de Jong and Lenske 1998, Krastev and Sammarruca 2006 for underlying theories), which easily allows to consider different neutron-proton composition of the neutron star matter, and also the inclusion of non-nucleonic degrees of freedom.

The full mean-field DBHF calculations of nuclear matter (Huber, Weber and Weigel 1995, Lee *et al.* 1998, Li, Machleidt, and Brockmann 1992) have been parameterized by Kotulič Bunta and Gmuca (2003), and we employ their parameterization with one-boson-exchange (OBE) potential A of Brockmann and Machleidt Li, Machleidt, and Brockmann (1992). We refer to the paper of Kotulič Bunta and Gmuca (2003) for the explicit set of values of the corresponding parameters. The model Lagrangian density includes the nucleon field  $\psi$ , isoscalar scalar meson field  $\sigma$ , isoscalar vector meson field  $\omega$ , isovector vector meson field  $\rho$ , and isovector scalar meson field  $\delta$ , including also the vector cross-interaction. The Lagrangian density in the form used by Kotulič Bunta and Gmuca (2003) reads

$$\begin{aligned} \mathcal{L}(\psi, \sigma, \omega, \rho, \delta) = & \bar{\psi}[\gamma_\mu(i\partial^\mu - g_\omega\omega^\mu) - (m_N - g_\sigma\sigma)]\psi \\ & + \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2) - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu \\ & - \frac{1}{3}b_\sigma m_N(g_\sigma\sigma)^3 - \frac{1}{4}c_\sigma(g_\sigma\sigma)^4 + \frac{1}{4}c_\omega(g_\omega^2\omega_\mu\omega^\mu)^2 \\ & + \frac{1}{2}(\partial_\mu\delta\partial^\mu\delta - m_\delta^2\delta^2) + \frac{1}{2}m_\rho^2\rho_\mu\rho^\mu - \frac{1}{4}\rho_{\mu\nu}\rho^{\mu\nu} \\ & + \frac{1}{2}\Lambda_V(g_\rho^2\rho_\mu\rho^\mu)(g_\omega^2\omega_\mu\omega^\mu) - g_\rho\rho_\mu\bar{\psi}\gamma^\mu\tau\psi + g_\delta\delta\bar{\psi}\tau\psi, \end{aligned} \quad (1)$$

where the antisymmetric tensors are

$$\begin{aligned} \omega_{\mu\nu} & \equiv \partial_\nu\omega_\mu - \partial_\mu\omega_\nu, \\ \rho_{\mu\nu} & \equiv \partial_\nu\rho_\mu - \partial_\mu\rho_\nu; \end{aligned} \quad (2)$$

the strength of the interactions of isoscalar and isovector mesons with nucleons is given by (dimensionless) coupling constants  $g$ 's and the self-coupling constants (also dimensionless) are  $b_\sigma$  (cubic),  $c_\sigma$  (quartic scalar) and  $c_\omega$  (quartic vector). The second and the fourth lines represent non-interacting Hamiltonian for all mesons,  $\Lambda_V$  is the cross-coupling constant of the interaction between  $\omega$  and  $\rho$  mesons. Furthermore,  $m_N$  is the nucleon mass,  $\partial^\mu \equiv \frac{\partial}{\partial x_\mu}$  and  $\gamma$ 's are the Dirac matrices (Kotulič Bunta and Gmuca 2003, Serot and Walecka 1986, Weber 1999).

We choose here three following parameterizations, which were shown to yield the best fits to the well-known properties of nuclear matter

*H* HA in Kotulič Bunta and Gmuca (2003) represents the best RMF fit to results obtained by Huber, Weber and Weigel 1995.

*L* LA in Kotulič Bunta and Gmuca (2003) represents the best RMF fit to results obtained by Lee *et al.* 1998, but does not include the  $\delta$  mesons to nucleons coupling.

*M* MA in Kotulič Bunta and Gmuca (2003) represents the best RMF fit to results obtained by Li, Machleidt, and Brockmann 1992, but does not include the  $\delta$  mesons to nucleons coupling.

The EoS of Kotulič Bunta and Gmuca which have been found to be a good description of asymmetric nuclear matter, are easily expressed up to about  $4 \times \rho_N$  (parameterization *H*) or even higher (parameterizations *L* and *M*).

## 2.2. $\beta$ -equilibrium

The total energy density of  $n$ - $p$ - $e$ - $\mu$  matter is given by

$$\mathcal{E} = \mathcal{E}_B(n_B, x_p) + \mathcal{E}_e(n_e) + \mathcal{E}_\mu(n_\mu), \quad (3)$$

where  $\mathcal{E}_B(n_B, x_p)$  is the binding energy density of asymmetric nuclear matter,  $n_i$  is the number density of different particles ( $i = n, p, e, \mu$ ),  $n_B = n_p + n_n$  is the baryon number density and  $x_p = n_p/n_B$  is the proton fraction. The leptonic contributions  $\mathcal{E}_l(n_l)$  ( $l = e, \mu$ ) to the total energy density are given by

$$\mathcal{E}_l(n_l) = \frac{2}{h^3} \int_0^{p_{F(l)}} (m_l^2 c^4 + p^2 c^2)^{1/2} 4\pi p^2 dp, \quad (4)$$

where  $p_{F(l)}$  is the Fermi momentum of  $l$ -th kind of particle.

The matter in neutron stars is in  $\beta$ -equilibrium, i.e. in equilibrium with respect to  $n \leftrightarrow p + e^- \leftrightarrow p + \mu$ . The (anti)neutrinos contribution could be neglected, because the matter is assumed to be cold enough that they can freely escape. The equilibrium is given by equality of chemical potentials  $\mu_n = \mu_p + \mu_e = \mu_p + \mu_\mu$ , where the chemical potential of each kind of particle is given by  $\mu_i = \partial \mathcal{E} / \partial n_i$ .

The chemical potentials of electrons and muons are simply  $\mu_l = \sqrt{m_l^2 c^4 + p_{F(l)}^2 c^2}$ , while the chemical potentials of nucleons are

$$\mu_{(p,n)} = \frac{\partial}{\partial n_{(p,n)}} (\mathcal{E}_B). \quad (5)$$

The binding energy density of asymmetric nuclear matter could be expressed in terms of proton fraction  $x_p$  (Danielewicz and Lee 2009)

$$\mathcal{E}_B(n_B, x_p) = \mathcal{E}_{\text{SNM}}(n_B) + (1 - 2x_p)^2 S(n_B), \quad (6)$$

where  $\mathcal{E}_{\text{SNM}}$  is the energy density of symmetric nuclear matter ( $x_p = 0.5$ ) and  $S(n_B)$  is the symmetry energy density, that corresponds to the difference of binding energy density between pure nuclear matter and symmetric nuclear matter

The symmetry energy  $S(n_B)$  is the factor corresponding to the second order term in expansion of binding energy density in terms of asymmetry parameter  $\delta = (n_n - n_p)/(n_n + n_p) = 1 - 2x_p$  and reads

$$S(n_B) = \frac{1}{2} \left. \frac{\partial^2 \mathcal{E}_B(n_B, \delta)}{\partial \delta^2} \right|_{\delta=0}. \quad (7)$$

From equation (6) one can see that symmetry energy is the difference of binding energy per particle between pure nuclear matter and symmetric nuclear matter.

$$S(n_B) = \mathcal{E}_B(n_B, x_p = 0) - \mathcal{E}_B(n_B, x_p = 0.5). \quad (8)$$

The condition of  $\beta$ -equilibrium then reads

$$\mu_e = \mu_\mu = \mu_n - \mu_p = 4 \frac{S(n_B)}{n_B} (1 - 2x_p). \quad (9)$$

and it is solved together with condition of charge neutrality ( $n_p = n_e + n_\mu$ ) to obtain the proton fraction of neutron star matter. The binding energy per baryon in dependence on the baryon number density is illustrated in Figure 1. The proton fraction of matter at the beta-equilibrium is given, for the chosen three EOS parameterizations, as a function of the baryon number density depicted in Figure 2.

### 2.3. EoS for low densities

The nuclear EoS have been the dominant input for the calculations in the high-density region, namely  $\rho \geq 10^{14} \text{ g/cm}^3$ . For lower densities, the EoS used are the following:

- Feynman-Metropolis-Teller EoS for  $7.9 \text{ g/cm}^3 \leq \rho \leq 10^4 \text{ g/cm}^3$  where matter consists of  $e^-$  and  ${}^{56}_{26}\text{Fe}$ , Feynman, Metropolis and Teller (1949);
- Baym-Pethick-Sutherland EoS for  $10^4 \text{ g/cm}^3 \leq \rho \leq 4.3 \times 10^{11} \text{ g/cm}^3$  with Coulomb lattice energy corrections Baym, Pethick, and Sutherland (1971);

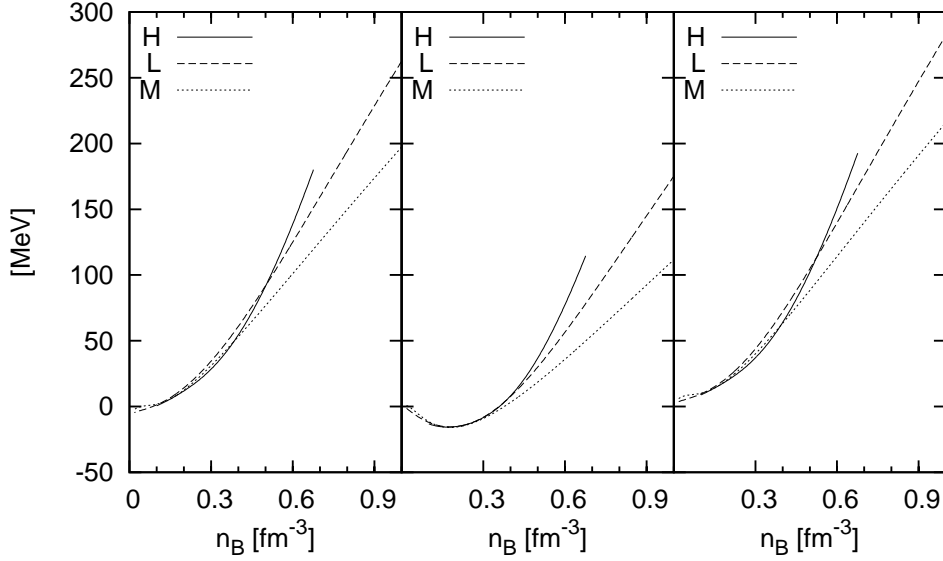


Figure 1: Binding energy per particle of different types of nuclear matter for used parameterizations. *Left* Matter at  $\beta$ -equilibrium, *Middle* symmetric nuclear matter, and *Right* pure neutron matter

- Baym-Bethe-Pethick EoS for  $4.3 \text{ g/cm}^3 \times 10^{11} \leq \rho \leq 10^{14} \text{ g/cm}^3$ : here,  $e^-$ , neutrons and equilibrated nuclei calculated using the compressible liquid drop model Baym, Bethe, and Pethick (1971).

### 3. Neutron star models

We consider static spherically symmetric models of neutron stars. The interior spacetime is described by the internal Schwarzschild metric (see, e.g., Misner, Thorne and Wheeler 1973, Haensel, Potekhin and Yakovlev 2007) that can be written in geometrical units ( $c = G = 1$ ) as

$$ds^2 = -e^{2\nu} dt^2 + e^{2\lambda} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (10)$$

where the radial component of metric can be expressed as a function of energy density  $\rho$

$$e^{2\lambda} = \frac{r}{r - 2m(r)}, \quad m(r) = 4\pi \int_0^r \rho r_1^2 dr_1. \quad (11)$$

The matter is assumed to be perfect fluid described by the energy momentum tensor

$$T^{\mu\nu} = (P + \rho)u^\mu u^\nu + P g^{\mu\nu}, \quad (12)$$

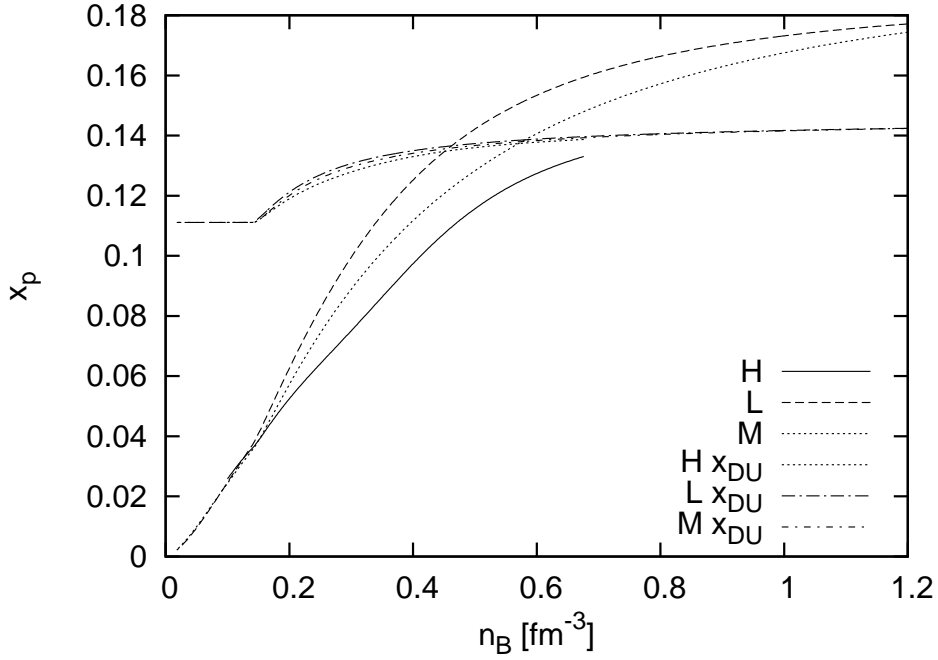


Figure 2: Proton fraction of matter being at  $\beta$  - equilibrium for used parameterizations. Also lines of direct URCA threshold (marked with  $x_{DU}$ ) for all parameterizations are depicted.

(Misner, Thorne, and Wheeler 1973). where  $P$  is the pressure,  $u^\mu$  is the 4-velocity of matter and  $g^{\mu\nu}$  is the metric tensor. The energy momentum tensor satisfies the conservation law  $T^{\mu\nu}{}_{;\nu} = 0$ .

The hydrostatic equilibrium is in general relativity given by the Tolman-Oppenheimer-Volkoff equation (TOV) (Oppenheimer and Volkoff 1939, Tolman 1939), which reads

$$\frac{dP}{dr} = -(\rho + P) \frac{m(r) + 4\pi r^3 P}{r(r - 2m(r))}. \quad (13)$$

Integration of TOV starting from given central energy density  $\rho_c$  uses the EoS and finally yields the radius  $R$ , given by the boundary condition  $P(R) = 0$ , and the gravitational mass  $M = m(R)$  of the neutron star.

Another useful quantity to calculate is the so-called baryonic mass  $M_B$  that represents the total number of baryons contained in the neutron star multiplied by the atomic mass unit  $u$ . The baryonic mass is then expressed as

$$M_B = 4\pi u \int_0^R n_B(r) \left[ 1 - \frac{2m(r)}{r} \right]^{-1/2} r^2 dr, \quad (14)$$

where  $n_B(r)$  is the baryon number density at the radius  $r$ .

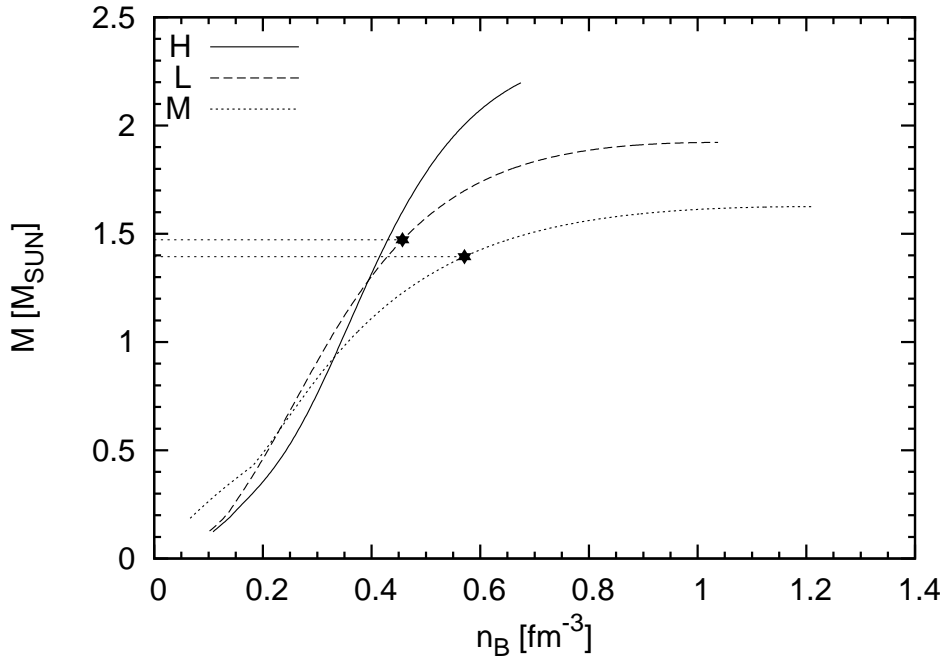


Figure 3: Mass given as a function of central baryon number density for different parameterizations. The stars correspond to the minimum mass of a neutron star that could cool via direct URCA reactions.

#### 4. Results versus observations

Several dozens of neutron stars and/or similar objects have their masses reported; a great majority of them is in very close vicinity of  $1.4 M_{\odot}$ , and only very few are significantly above (see, e.g., the compilations in Bethe, Brown and Lee (2007), Lattimer and Prakash 2007) and observations and analyses (see, e.g, Rikovska Stone *et al.* 2003, Weber, Negreiros and Rosenfeld 2007, Podsiadlowski *et al.* 2005, Trümper *et al.* 2004, Pons *et al.* 2002, Kramer and Wex 2009, Krastev and Sammaruca 2006, Lattimer and Prakash 2007, Blaschke, Klähn and Sandin 2008, Dexheimer, Vasconcellos and Bodmann 2008, Klähn *et al.* 2006, Nice, Stairs and Kasian 2008, Rikovska Stone *et al.* 2007). However, recent results of the data fitting of kHz quasiperiodic oscillations observed in the low-mass X-ray systems containing neutron stars indicate relatively high masses of  $M > 2 M_{\odot}$  (Belloni, Mendez and Homan 2007, Török *et al.* 2008a,b,c, Barret, Olive and Miller 2005, Boutelier *et al.* 2010, Boutloukos *et al.* 2006) which could provide very strong constraint on the EoS. On the other hand, modification of the characteristic orbital frequencies by a magnetic repulsion caused by the interaction of slightly charged matter in accretion disc in vicinity of a neutron star with dipole magnetic field could

shift the mass estimates to lower values close to canonical  $1.4 M_{\odot}$  (Bakala *et al.* 2008). Our calculations with parameterization *H* allow for the existence of neutron stars even for so heavy masses.

#### 4.1. Direct URCA constraints

The proton fraction  $x_p$  of matter in  $\beta$  equilibrium is presented in Figure 2 together with the direct URCA threshold. The direct URCA reactions  $n \rightarrow p + e^- + \bar{\nu}_e$  could operate only if the proton fraction exceeds the threshold given by the condition

$$x_{\text{DU}} = \frac{1}{1 + \left(1 + x_e^{1/3}\right)^3}, \quad (15)$$

where  $x_e = n_e/(n_e + n_\mu)$ . One can see that only parameterizations *L* and *M* enable rapid cooling. The threshold densities are  $n_{\text{DU}} = 0.457 \text{ fm}^{-3}$  in the case of parameterization *L* and  $n_{\text{DU}} = 0.571 \text{ fm}^{-3}$  in the case of parameterization *M*. These values correspond (see Figure 3) to neutron star masses  $M = 1.47 M_{\odot}$  (parameterization *L*) and  $M = 1.39 M_{\odot}$  (parameterization *M*). Parameterizations *L* and *M* thus do not fulfill the direct URCA constraints, however Klähn *et al.* 2006 used also the value  $1.35 M_{\odot}$  as a weaker test that is passed also by parameterizations *L* and *M*.

#### 4.2. Maximum mass

The maximum mass limit is probably the most often used test of the equation of state. The maximum masses given by EoS used in this paper are  $M_{\text{max}}^H = 2.18 M_{\odot}$ ,  $M_{\text{max}}^L = 1.92 M_{\odot}$  and  $M_{\text{max}}^M = 1.62 M_{\odot}$ . The maximum mass obtained for objects containing matter described by parameterizations *L* and *M* follows the requirements of stability with respect to radial oscillations ( $\partial M/\partial n_c > 0$ ). In the case *H* we used the values corresponding to the central density  $n_B(r=0) = 0.66 \text{ fm}^{-3}$ , because for the densities above this value the model used for the EoS is not without questions and also because only with central densities up to about  $4 \times$  normal nuclear density we were able to explain masses of neutron stars that meet the observational requirements. With some extrapolations, higher masses could be in principle modelled, but we decided to use parameterization *H* up to the quoted density only since there is no current astrophysical observation of such a high mass. The observation of high mass is however crucial and very promising issue of astrophysical observations. It should be noted that Klähn *et al.* 2006 used the value that could not be used anymore. Also the result for the source 4U 1636–536 that gives  $M = (1.9 - 2.1) M_{\odot}$  as proposed by Barret, Olive and Miller (2005) should be used rather as an upper limit of the neutron star mass than as its estimate, see, e.g. Miller, Lamb and Psaltis (1998) for underlying theories. The neutron star mass is inferred due to the highest observed frequency of QPOs observed in the system, under the assumption of identifying the highest frequency with the Keplerian frequency of the innermost stable circular orbit (ISCO). Clearly this gives an upper limit on the

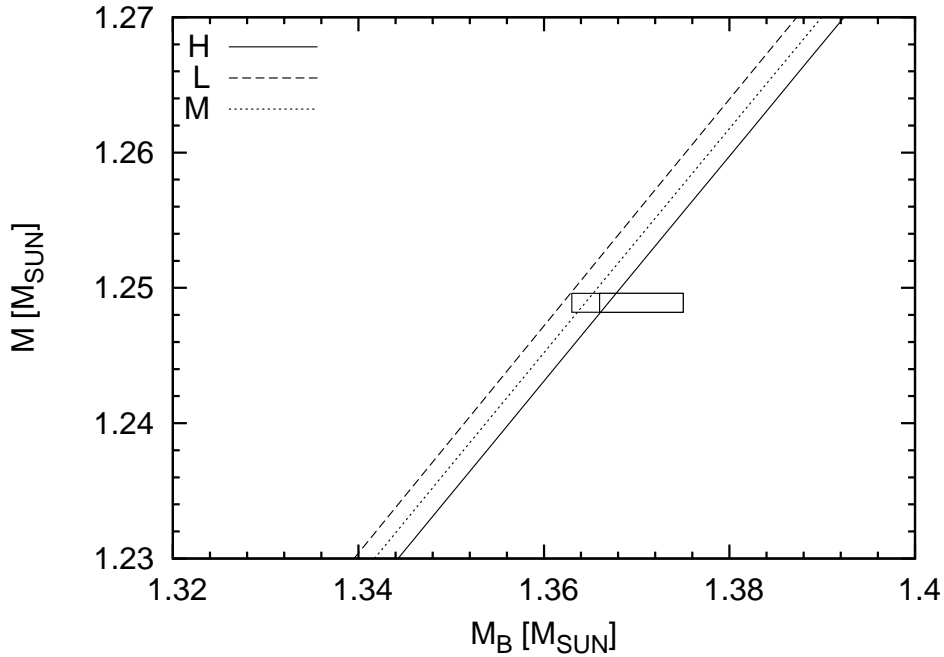


Figure 4: Relation of calculated gravitational mass  $M$  and the baryonic one  $M_B$  for different parameterizations. The limitations imposed by the analysis of the J0737-3039 double pulsar are drawn as a small rectangle. The box is also extended to the left by  $0.003M_\odot$  indicating the possible mass loss.

mass, and the real neutron star mass has to be expected smaller because the QPOs have to be excited above ISCO. Up to date, one of the two pulsars Ter 5 I and J has a reported mass larger than  $1.68 M_\odot$  to 95% confidence level (see, e.g. Lattimer and Prakash (2007) and references therein)<sup>1</sup>. Champion *et al.* (2008) predicted mass of PSR J1903+0327 to be  $M = 1.74 \pm 0.04 M_\odot$ . Freire (2009) estimated the mass for the same source to be  $M = 1.67 \pm 0.01 M_\odot$ . These values, even if they are different, give approximately the same limit on mass when they are combined together, namely  $\gtrsim 1.66 M_\odot$  at  $2 \sigma$  level. These predictions are not in favour the parameterization  $M$  with  $M_{\max}^M = 1.62 M_\odot$ .

#### 4.3. Double pulsar J0737–3039

Podsiadlowski *et al.* (2005) investigated possible formation scenarios of double pulsar J0737–3039. They have shown that one can test EoS assuming the pulsar B is formed by an electron-capture supernova. Such scenario enables formation of the

<sup>1</sup>The individual pulsar masses unfortunately are not assumption-independent. In our discussion, we adhere to the value  $1.68 M_\odot$  reported by Lattimer and Prakash, but bearing in mind the possible uncertainty in its derivation.

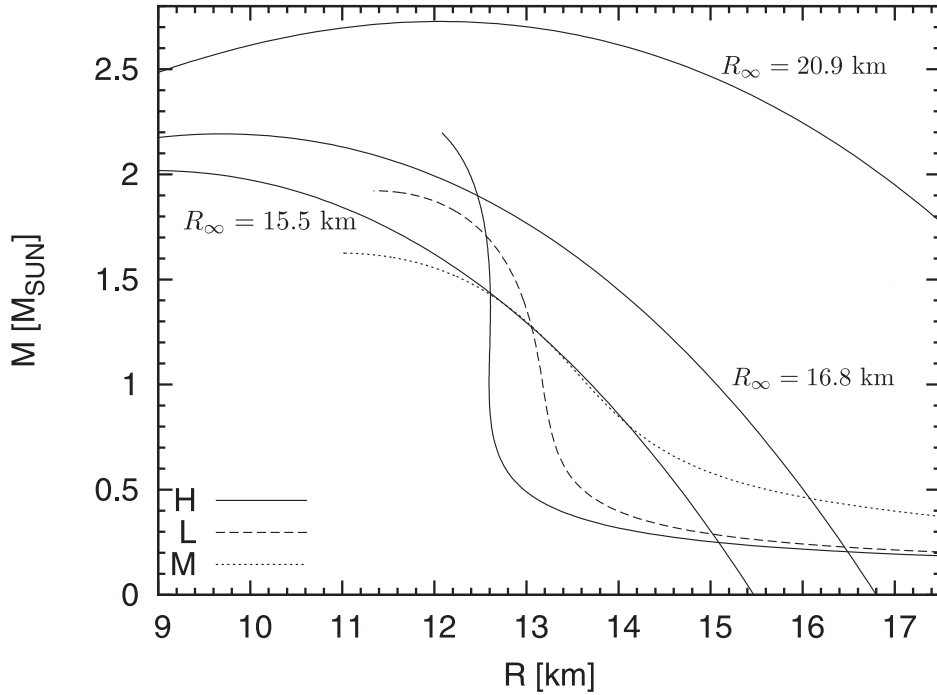


Figure 5: Mass–radius relation for different parameterizations. The lines corresponding to RX J 1856.5–3754 gives a lower mass limit, that should the given EoS get over.

pulsar B that has low but very accurately measured mass  $M = 1.2489 \pm 0.0007 M_{\odot}$  (Kramer and Wex 2009). If this pulsar is born under the presented scenario, its baryonic mass  $M_B$  should be in the range 1.366 to 1.375  $M_{\odot}$ . The authors also argue the matter loss being low (the matter loss they give is few times  $10^{-3} M_{\odot}$ ). The relation between the gravitational and the baryonic masses together with the limitations derived from the double pulsar observations are presented in Figure 4. One can see that the only parameterization that meets requirements assuming no mass loss is the parameterization *H*. The parameterization *M* is able to explain the results if one includes mass loss predicted by Podsiadlowski *et al.* (2005). Unfortunately this parameterization was ruled out by the maximum mass test.

#### 4.4. Isolated neutron star RX J1856.5–3754

Several authors (see, e.g., Trümper *et al.* 2004, Pons *et al.* 2002, van Kerkwijk and Kaplan 2007, Steiner *et al.* 2010) discussed observations of the isolated neutron star RX J1856.5–3754 and they found constraints on the mass-radius relation of this particular neutron star. They found the limits of the apparent radius being given by

the mass-radius relation

$$\frac{M}{M_{\odot}} = \frac{R}{2.95 \text{ km}} \left( 1 - \frac{R^2}{R_{\infty}^2} \right), \quad (16)$$

that could serve as a test of equation of state. We have used three different values for  $R_{\infty}$  namely  $R_{\infty} = 15.5, 16.8, 20.9$  km. None of tested parameterizations is able to explain the apparent radius  $R_{\infty} = 20.9$  km. Parameterization  $H$  is the only one capable of explaining the apparent radius  $R_{\infty} = 16.8$  km estimated by Trümper *et al.* (2004). The lowest predicted apparent radius could be modeled by all parameterizations considered in this paper. The mass–radius relations for all parameterizations together with observational limits are illustrated in Figure 5.

## 5. Conclusions

We have employed the parameterized form of the relativistic mean-field EoS for asymmetric nuclear matter with vector cross interaction. The proton fraction was varied in accord with the need of the  $\beta$ -equilibrium and charge neutrality. Assuming spherically symmetric geometry and using TOV equation, we constructed models of neutron stars for different central parameters. We have used set of observational data to test EoS of nuclear matter represented by three different parameterizations of relativistic Brueckner-Hartree-Fock equation.

We have shown that only the parameterization  $H$  is able to pass almost all the tests considered in this paper. The only exception is the apparent radius  $R_{\infty} = 20.9$  km estimation for the isolated neutron star RX J1856.5-3754; however this estimate is based on distance measurements being still widely discussed. This parameterization also represents the only EoS based on the relativistic Brueckner-Hartree-Fock theory that could explain the formation of pulsar B in the double pulsar system J 0737–3039 without mass loss.

Our present calculations have been done considering only neutrons and protons in  $\beta$ -equilibrium with electrons and muons. We aim to continue in tests of given EoS in future. One of our plans is to include hyperons. Another is to perform more detailed tests based on the promising fitting of observational data of quasi-periodic oscillations in low-mass X-ray systems measurements. This necessitates to investigate the rotational effects on neutron star models based on the Hartle-Thorne metric reflecting mass, spin and the quadrupole moment of the neutron star (Hartle 1967, Hartle and Thorne 1968). Our preliminary results indicate that these improvements could bring a new information on the validity of EoS (Stuchlík *et al.* 2007). The important role of the neutron star spin is demonstrated in the case of Circinus X–1 (Török *et al.* 2010).

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