Astronomy & Astrophysics manuscript no. 2558 July 4, 2005
(DOI: will be inserted by hand later)

A possible 3:2 orbital epicyclic resonance in QPO frequencies of Sgr A*

Gabriel Török[1,2,3]

[1] Nordita, Blegdamsvej 17, DK-2100 Copenhagen, Denmark
[2] Department of Astrophysics, Göteborg University, & Theoretical Physics, Chalmers University, S-412-96 Göteborg, Sweden
[3] Institute of Physics, Faculty of Philosophy and Science, Silesian University in Opava, Bezručovo nám. 13, CZ-746 01 Opava, Czech Republic
e-mail: terek@volny.cz

Received . . . ; accepted . . .

Abstract. A recent measurement of double peak QPOs frequencies in Sgr A* is consistent with the 3:2 ratio. The same ratio is firmly established by previous observations in all double peaked kHz QPOs in microquasars and theoretically explained by orbital epicyclic resonances excited in nearly Keplerian accretion flow in a black hole's strong gravity. If confirmed, the 3:2 ratio of double peak QPOs in Sgr A* will be of a fundamental importance for the black hole accretion theory, by providing another clear argument that the accretion disk oscillations are indeed governed by non-linear, strong-gravity physics.

Key words. black holes – X-ray variability – Sgr A* – observations – theory

1. Double peak QPOs with the 3:2 ratio in Sgr A* ?

From the current analysis of stellar orbits within 10-100 light hours of Sgr A*, obtained independently by the MPI Garching group (Schoedel et al., 2002, 2003; Eisenhauer et al., 2003) and the UCLA group (Ghez et al., 2003, 2004) the best estimate of the black hole central mass is $3.6\pm0.4\times10^6\ M_\odot$, where the error bars represent both statistical and systematic errors. Earlier lower statistical mass estimates based on proper motions of stars further away gave somewhat lower masses ($2.6\times10^6\ M_\odot$) but in light of new information on stellar distribution and anisotropies these earlier data would now also lead to masses near $3.5\times10^6\ M_\odot$ (see the discussion in Schoedel et al., 2003). This well constrained mass must be contained within a few light hours, i.e. several hundred Schwarzschild radii. The analysis of the spatial distributiuon of the stellar cusp centered on the BH suggests that most likely no more than $1\times10^3\ M_\odot$ of that is in the form of stars or stellar remnants (the latter is less well constrained: Genzel et. al., 2003). From the lack of motion of the radio source itself and a theoretical comparison of the stochastic motions of a BH of different masses with surrounding stars, a lower limit of the mass contained within the radius of the radio source (10 light minutes, 20 Schwarzschild radii) is about $\sim 10^5 M_\odot$ (Reid et al., 1999, 2003; Backer & Sramek, 1999; Schoedel et al., 2003).

From these measurements and discussion, one concludes that the mass of the black hole in Sgr A* is most likely in the interval

$$2.6\times10^6 M_\odot < M < 4.4\times10^6 M_\odot, \tag{1}$$

and that a very conservative lower limit is $\sim 10^5 M_\odot$[1].
Genzel et. al. (2003) measured a clear periodicity of 17 min (1020 sec) in Sgr A* variability during a flaring event. This period is in the range of Keplerian orbital periods at a few gravitational radii away from a black hole with the mass constrained by (1). More recently, Aschenbach et al. (2004) have reported three other QPOs periodicities, 692 sec, 1130 sec, 2178 sec, roughly in the orbital Keplerian range, and two much shorter periods of 100 sec and 219 sec. The value of 1130 sec differ by 10% from the 1020 sec period found by Genzel et. al. (2003) and may correspond to the same periodicity of the source, but a firm conclusion is not possible because of the quality of the data. With all reservations and caution that are necessary here, it was noticed (Abramowicz et al., 2004a,c; Aschenbach, 2004) that

$$(1/692) : (1/1130) : (1/2178) \approx 3 : 2 : 1, \tag{2}$$

[1] We thank Reinhard Genzel for providing (in May 2004) this updated discussion on the Sgr A* mass measurements.

i.e. that the "Keplerian" frequencies found in Sgr A* form ratios that are very close to be an exact commensurable sequence, 3:2:1. The commensurability of QPOs frequencies in Sgr A*, if confirmed by a more accurate observations and data analysis, could be of a fundamental importance for a reason that we explain in the next section.

2. Commensurability of QPOs in microquasars: observations and theory

In the case of stellar mass black hole sources, impressively accurate ratio of frequencies $\nu_{\rm upp}/\nu_{\rm down} = 3/2$ was found in all four microquasars that display the double peak QPOs. Table 1 and Figure 1 summarize the QPO microquasar data relevant to the present Note. In three microquasars with known mass M, the QPOs frequencies scale as (McClintock & Remillard, 2003),

$$\nu_{\rm upp} = 2.8 \left(\frac{M}{M_{\odot}}\right)^{-1} [\mathrm{kHz}]. \tag{3}$$

Table 1. Frequencies of twin peak QPOs in microquasars and Galaxy centre black hole

Source [(a)]	$\nu_{\rm upp}$ [Hz]	$\Delta\nu_{\rm upp}$ [Hz]	$\nu_{\rm down}$ [Hz]	$\Delta\nu_{\rm down}$ [Hz]	$2\nu_{\rm upp}/3\nu_{\rm down} - 1$	Mass [(b)] [$M_{\odot}$]
GRO 1655–40	450	± 3	300	± 5	0.00000	6.0 — 6.6
XTE 1550–564	276	± 3	184	± 5	0.00000	8.4 — 10.8
H 1743–322	240	± 3	166	± 8	-0.03614	not measured
GRS 1915+105	168	± 3	113	± 5	0.00885	10.0 — 18.0
Sgr A*	1.445	±0.16 mHz	0.886	±0.04 mHz	0.08728	2.6 — 4.4 10^6

[(a)] Twin peak QPOs first reported by Strohmayer (2001); Remillard, Muno, McClintock & Orosz (2002); Homan et al. (2003); Remillard, Muno, McClintock & Orosz (2003); Aschenbach et al. (2004).
[(b)] See Greene, Bailyn, Orosz (2001); Orosz et al. (2002); Greiner, Cuby, McCaughrean (2001); McClintock & Remillard (2003), and the first par of introduction.

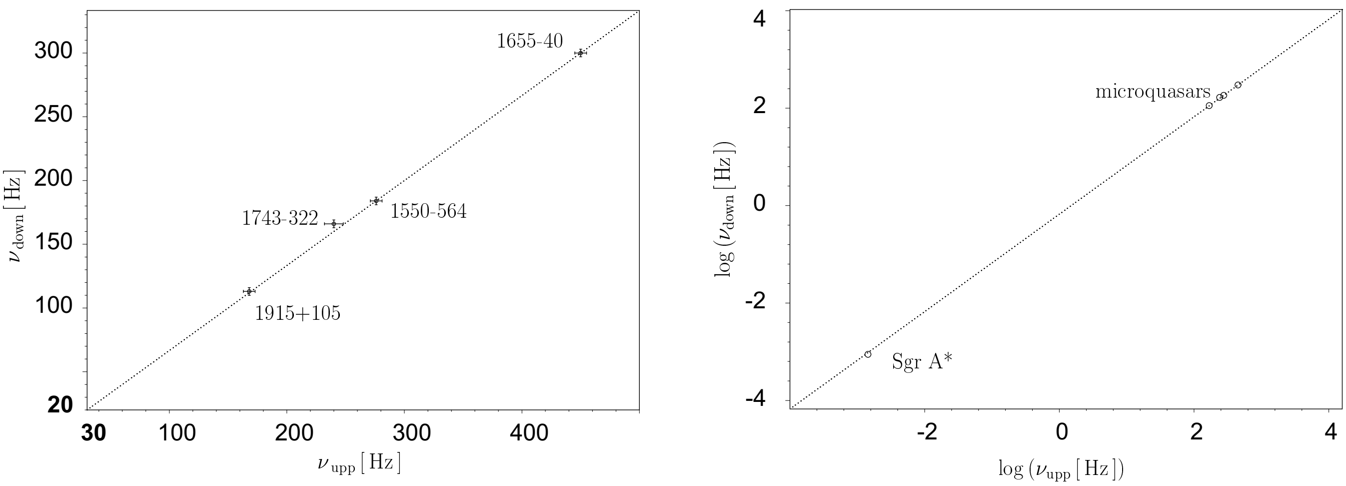

Fig. 1. *Left:* In all four microquasars where double peak kHz QPOs were detected, the observed frequencies $\nu_{\rm upp}$ and $\nu_{\rm low}$ are clearly in 3:2 ratio. *Right:* The same 3:2 ratio seems to be present in double peak QPOs in Sgr A*. The accuracy is so high that the error bars cannot be shown correctly in this logarithmic plot.

Even before the double peak kHz QPOs were discovered in microquasars (first by Strohmayer, 2001), and the 3:2 ratio pointed out (first by Abramowicz & Kluźniak, 2001), Kluźniak & Abramowicz (2000) suggested on theoretical ground that these QPOs should have rational ratios, being due to resonances in oscillations of nearly Keplerian accretion disks. It seems that the resonance hypothesis is now well supported by observations, and that in particular the 3:2 ratio is seen most often in double peak QPOs in LMXB black hole and neutron sources, $2\nu_{\rm upp} = 3\nu_{\rm down}$.

According to the resonance hypothesis, the two modes in resonance have eigenfrequencies $\nu_{\rm rad}$, equal to the radial epicyclic frequency, and $\nu_{\rm v}$, equal to the vertical orbital frequency $\nu_{\rm vert}$ or the Keplerian frequency $\nu_{\rm K}$ (see Kluźniak & Abramowicz, 2004, for a recent rewiev). Several resonances of this kind are possible, and have been discussed (see e.g. Abramowicz et al., 2004b). Main relations are summarized in Table 2.

Table 2. Relation of observed frequencies for standard ($\nu = \nu_{\rm vert}$) and "Keplerian" ($\nu = \nu_{\rm K}$) resonances

Theory				Observed frequencies	
	Type of resonance	$n\nu_{\rm rad} = m\nu$			
		n	m	$\nu_{\rm upp}$	$\nu_{\rm down}$
standard	parametric	3	2	$\nu_{\rm vert}$	$\nu_{\rm rad}$
	3:1 forced	3	1	$\nu_{\rm vert}$	$\nu_{\rm vert} - \nu_{\rm rad}$
	2:1 forced	2	1	$\nu_{\rm vert} + \nu_{\rm rad}$	$\nu_{\rm vert}$
Keplerian	parametric	3	2	$\nu_{\rm K}$	$\nu_{\rm rad}$
	3:1 forced	3	1	$\nu_{\rm K}$	$\nu_{\rm K} - \nu_{\rm rad}$
	2:1 forced	2	1	$\nu_{\rm K} + \nu_{\rm rad}$	$\nu_{\rm K}$

Formulae for $\nu_{\rm vert}$ and $\nu_{\rm rad}$ in the gravitational field of a rotating Kerr black hole with the mass M and spin a are well known,

$$\nu^2_{\rm vert} = \nu^2_{\rm K}\left(1 - 4\,a\,x^{-3/2} + 3a^2\,x^{-2}\right), \quad \nu^2_{\rm rad} = \nu^2_{\rm K}\left(1 - 6\,x^{-1} + 8\,a\,x^{-3/2} - 3\,a^2\,x^{-2}\right), \quad \nu_{\rm K} = \frac{1}{2\pi}\left(\frac{GM_0}{r_G^3}\right)^{1/2}\left(x^{3/2} + a\right)^{-1}. \tag{4}$$

Here $x = r/(GM/c^2)$ is the dimensionless radius, expressed in terms of the gravitational radius of the black hole.

For a particular resonance n:m, the equation $n\nu_{\rm rad} = m\nu$ ($\nu = \nu_{\rm vert}$ or $\nu_{\rm K}$) determines the dimensionless resonance radius $x_{\rm n:m}$ as a function of a.

3. Application to Sgr A*

From the known mass of Sgr A*, the observed $\nu_{\rm down} = 0.886\,{\rm mHz} = 1/1130\ {\rm sec}^{-1}$, and from equation (4) one may calculate the black hole spin in SgrA*, consistent with different types of resonances. This procedure was first applied to the microquasar GRO 1655–40 by Abramowicz & Kluźniak (2001) and more recently for the other two microquasars by Abramowicz et al. (2004b) and Török et al. (2005). These results, together with these calculated in this Note for five representative values of the Sgr A* mass, are summarized in Table 3, and illustrated in Figure 2 for particular resonances.

Table 3. Sgr A* spin estimates from observed 3:2 QPOs, calculated for five representative values of mass from a large spectrum above the lower conservative limit including the best mass estimate $3.6 \times 10^6 M_\odot$

Resonance	Mass [$M_\odot$] :						
	$4.4\,10^5$	$0.8\,10^6$	$1.8\,10^6$	$2.2\,10^6$	$2.6\,10^6$	$3.6\,10^6$	$4.4\,10^6$
3:2 $[\nu_\theta,\ \nu_r]$ parametric	—	0.22	0.90	0.98	—	—	—
2:1 $[\nu_\theta,\ \nu_r]$ forced	—	—	0.16	0.40	0.57	0.81	0.92
3:1 $[\nu_\theta,\ \nu_r]$ forced	—	—	0.36	0.58	0.72	0.95 (0.99)*	—
3:2 $[\nu_{\rm K},\ \nu_r]$ "Keplerian" p.	—	0.25	—	—	—	—	—
2:1 $[\nu_{\rm K},\ \nu_r]$ "Keplerian" f.	—	—	0.16	0.41	0.58	0.83	0.94
3:1 $[\nu_{\rm K},\ \nu_r]$ "Keplerian" f.	—	—	0.32	0.52	0.65	0.85	0.93

* see Figure 2.

4. Discussion and conclusions

If commensurability of double peak QPOs frequencies in Sgr A* is confirmed, this together with the already established $1/M$ scaling [2] , would give very strong support for the suggestion that the double peak QPOs physics, in microquasars and in Sgr A*,

[2] See McClintock & Remillard (2003); Kluźniak et al. (2004).

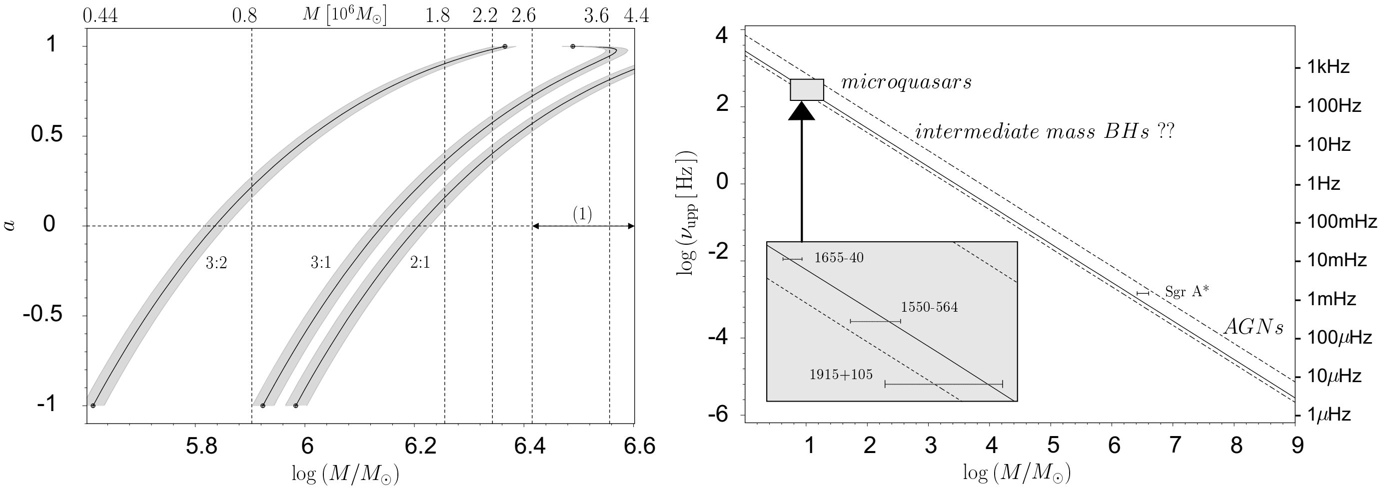

Fig. 2. *Left:* Spin dependence for 3:2 parametric, 3:1 and 2:1 forced resonance in Sgr A* implied by measured $\nu_{\rm down} = 0.886\,{\rm mHz}$, shadows respect accuracy of measuring. *Right:* The case of the 2:1 parametric resonance for observed frequencies. Mass range for Sgr A* corresponds to the range given by inequality (1). Dotted lines are for $a = 0$ (lower line) and $a = 1$ (upper line), the solid line is the best fit $\nu_{\rm upp}$ vs. $1/M$ found by McClintock & Remillard (2003) and described by equation (3).

is due to a non-linear orbital resonance in strong gravity. It would be interesting to see whether other black hole sources, ULXs and AGNs, show the same phenomenon (Abramowicz et al., 2004a).

For black hole sources with known mass that display double peak QPOs, one may measure the black hole spin, but the spin estimate depends on which of the theoretically possible resonances, 2:1, 3:1, or 3:2, is actually excited in the source. At present, neither observations, nor the resonance theory can firmly determine this[3].

Acknowledgements. I thank Marek Abramowicz and Wlodek Kluzniak and also Zdenek Stuchlik and Vladimir Karas for discussion and help. The article was partialy written under the Erasmus-Socrates exchange program between Chalmers University and Silesian University at Opava. The final version was realized with the suppport of Czech grant MSM 4781305903 and completed at Nordita (Copenhagen).

References

Abramowicz M.A., Kluźniak W., 2001, A&A 374L,19A

Abramowicz M.A., Kluźniak W., McClintock & Remillard, 2004a, ApJ 609, L63

Abramowicz M.A., Kluźniak W., Stuchlík Z., Török G., 2004b, submitted to A&A, astro-ph/0401464

Abramowicz M.A., Kluźniak W., 2004, X-Ray Timing 2003: Rossi and Beyond, ed. P. Kaaret, F. K. Lamb, J. H. Swank (Melville, NY: American Institute of Physics)

Abramowicz M.A., Kluźniak W., Stuchlík Z., Török G., 2004c, In S. Hledík and Z. Stuchlík, editors, Proceedings of RAGtime 4/5: Workshops on black holes and neutron stars, Opava, 1416/1315 October 2002/03, Opava, 2004. Silesian University at Opava.

Aschenbach B., Grosso N., Porquet D., Predehl P., 2004, accepted by the A&A, astro-ph 040158

Aschenbach B., 2004, accepted by A&A, astro-ph 0406545

Backer D. C., Sramek R. A., 1999, ApJ, 524, 805

Eisenhauer et al. 2003, ApJ 597, L121

Genzel et al. 2003, ApJ 594, 812

Ghez et al. 2003, ApJ 586, L127

Ghez et al. 2004, astro-ph 0306130

Greene J., Bailyn Ch. D., Orosz J. A., 2001, ApJ, 554, 1290G

Greiner J., Cuby J. G., McCaughrean M. J., 2001 Natur., 414, 522G

Homan J., Miller J.M., Wijnands R., Steeghs D., Belloni T., van der Klis M., Lewin W. H. G., 2003, Atel 16 http://integral.rssi.ru/atelmirror/

Kluźniak W., Abramowicz M.A., 2000, Phys. Rev. Lett. submitted, astro-ph/0105057

[3] Aschenbach (2004) argues that QPOs data suggest that black holes in the three microquasars and Sgr A* listed in Table 1 have nearly the same spin $a \approx 0.99616$, due to a new relativistic effect that he found (a non monotonic behaviour of orbital velocity with radius for rapidly rotating black holes, see also Stuchlik et al., 2004). Our calculations are based on standard types of non-linear orbital resonances, and do not include the Aschenbach effect.

Kluźniak W., Abramowicz M.A., 2003, 12th Workshop on General Relativity and Gravitation, Tokyo: Tokyo University Press, astro-ph/0304345

Kluźniak W., Abramowicz M.A., 2004, in "X-Ray Timing 2003: Rossi and Beyond", ed. P. Kaaret, F. K. Lamb, J. H. Swank (Melville, NY: American Institute of Physics)

Kluźniak W., Abramowicz M.A., Lee, W. H., 2004, in "X-Ray Timing 2003: Rossi and Beyond", ed. P. Kaaret, F. K. Lamb, J. H. Swank (Melville, NY: American Institute of Physics)

McClintock J.E, Remillard R.A., 2003, astro-ph/0306213 v.2

Orosz J. A., Groot P. J., van der Klis M., McClintock J. E., Garcia M. R., Zhao P., Jain R. K., Bailyn Ch. D., Remillard R. A., 2002, ApJ, 568, 845O

Remillard, R. A., Muno, M. P., McClintock, J. E., Orosz, J. A. 2002, ApJ, 580, 1030

Remillard, R. A., Muno, M. P., McClintock, J. E., Orosz, J. A. 2003, HEAD....7.3003R

Reid at al. 1999, ApJ, 524, 816

Reid et al. 2003, Astron. Nachr./AN 324, No. S1, 3 9 2003, astro-ph/0304095

Schoedel et al. 2002, Nature 419, 694

Schoedel et al. 2003, ApJ 596, 1015

Stuchlik, Z., Slany, P., Török, G., Abramowicz M.A., 2004, Phys.Rev.D., submitted, gr-qc/0411091

Strohmayer T., 2001, ApJ 552L, 49S

Török G., Abramowicz M.A., Kluźniak W., Stuchlík Z., 2005, submitted to A&A, astro-ph/0401464